

$a \cdot w \cdot b$ Harcanan enerji
 $F(a) \cdot w \cdot q$
 $\hookrightarrow F(a) \cdot (b-a) \cdot q$

$\Gamma = \{ \gamma(t) : 0 \leq t \leq 1, \gamma \in \mathbb{R}^n \}$ bir eğri
 $\gamma: e'$ olsun. $F: \mathbb{R}^n \rightarrow \mathbb{R}^n$ bir e'
 vektör alanı olsun.

2.2 Vektör Alanları.

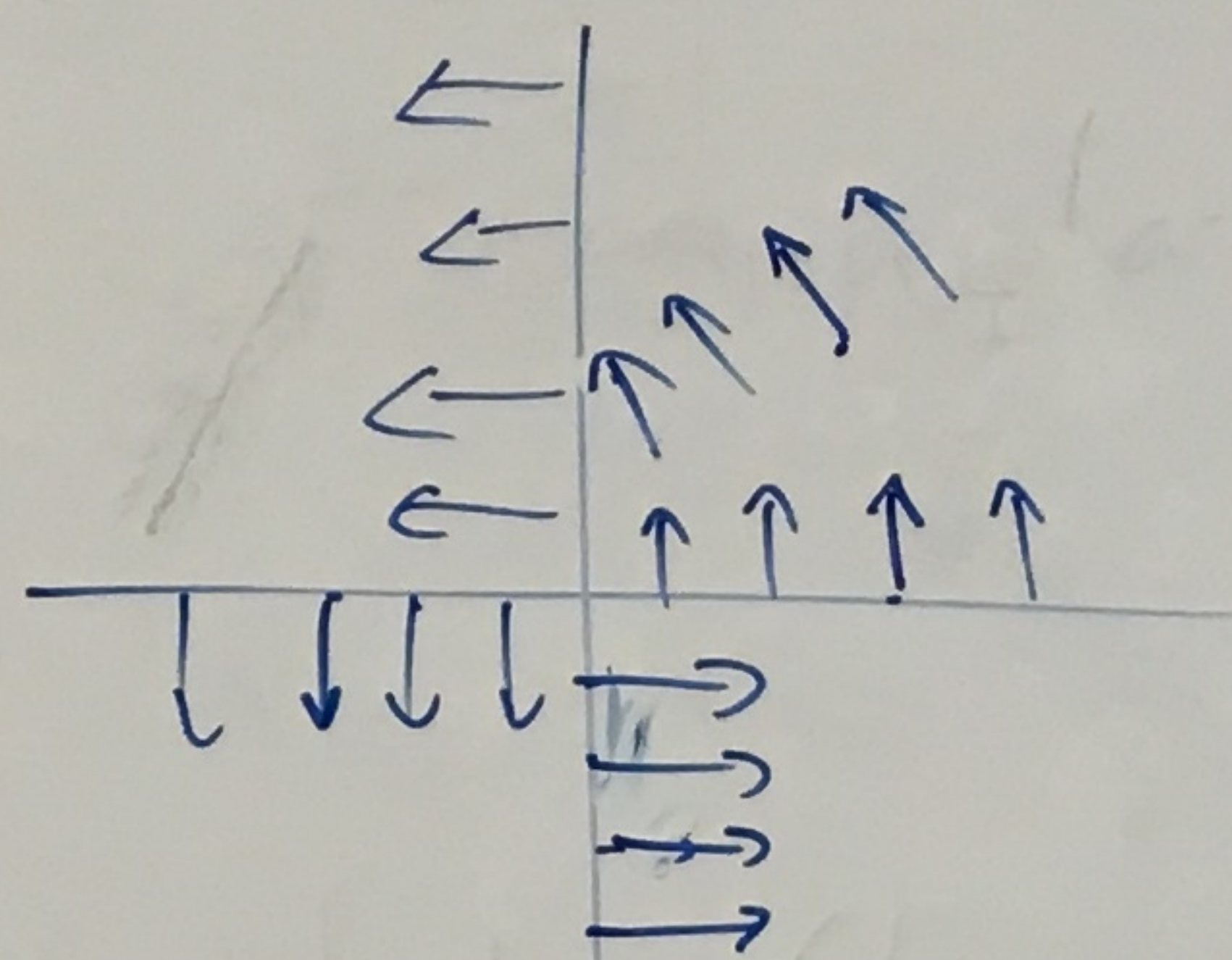
$$F: \mathbb{R}^n \rightarrow \mathbb{R}^n \quad \hookrightarrow$$

her $a \in \mathbb{R}^n$ için

$F(a)$ bir vektör olarak
düşünebiliriz.

Örnek 0.2.1 $F(x, y) = \left(\frac{y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right)$
 $(x, y) \neq (0, 0)$

$$\hookrightarrow F(r, \theta) = (\sin \theta, \cos \theta)$$



Soru

$F(x, y): \mathbb{R}^2 \rightarrow \mathbb{R}^2$ bir elektrik
 alanı olsun. q bir +ve yük
 olsun. İlk olarak q 'a' noktasında
 olsun. γ bir eğri Γ üzerinde
 hareket edecek. q 'a' noktadan
 'b' noktaya götürmek için ne kadar

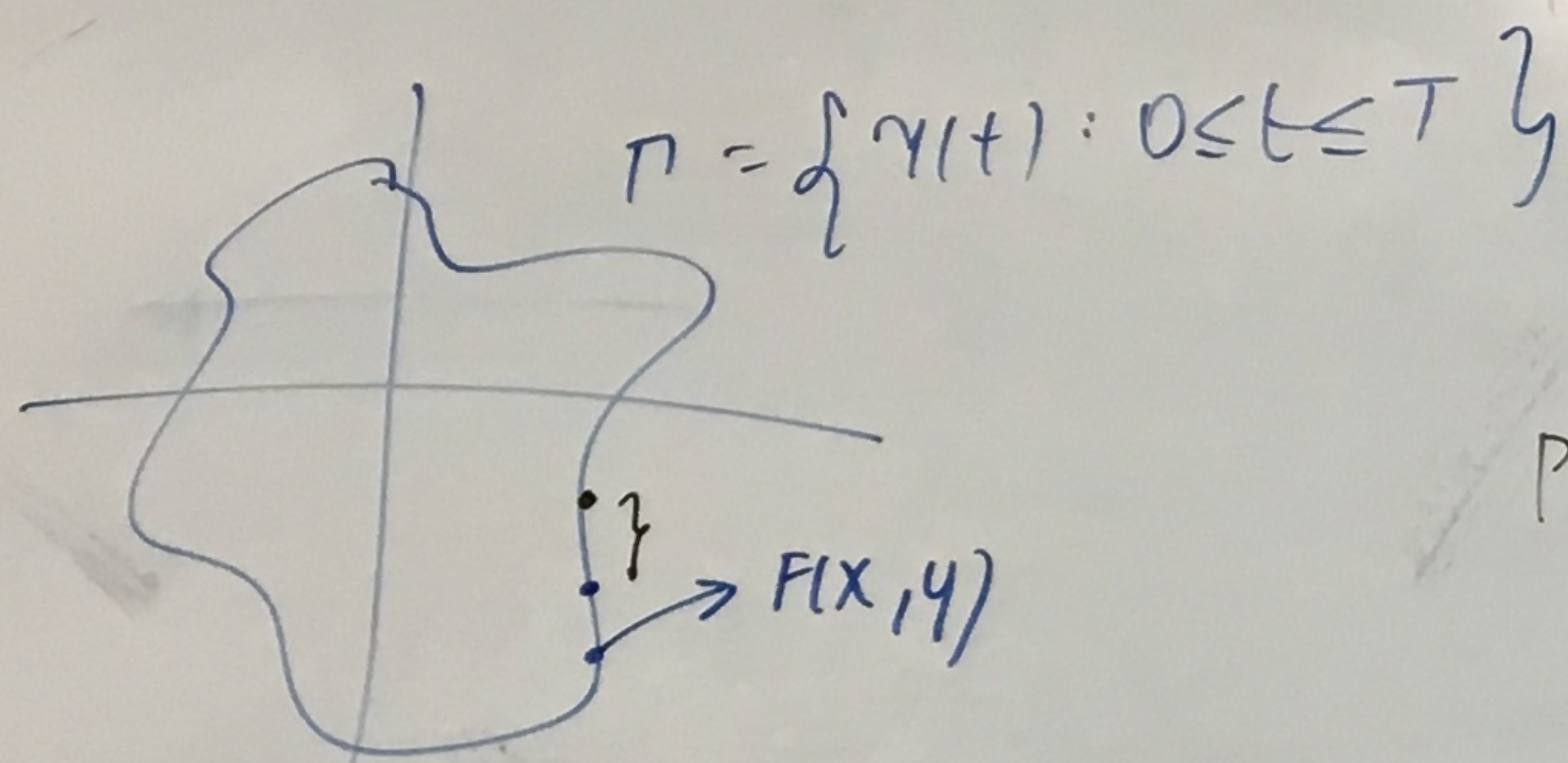
Enerji
harcanmış

her $a \in \mathbb{R}^n$ için $F(a)$ bir vektör olarak düşünürüz.

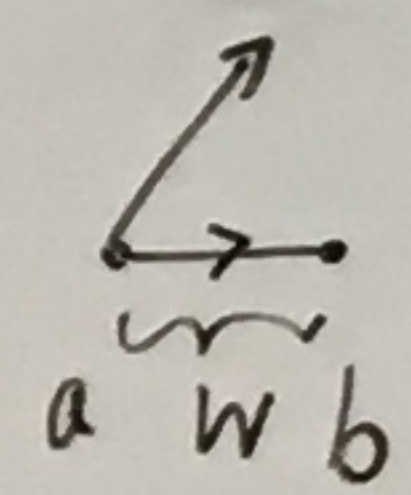
Örnek 0.2.1 $F(x, y) = \left(\frac{y}{x^2+y^2}, \frac{-x}{x^2+y^2} \right)$
 $(x, y) \neq (0, 0)$

$\hookrightarrow F(r, \theta) = (\sin \theta, -\cos \theta)$

Soru $F(x, y) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ bir elektrik alan olsun. q bir +ve şarj olsun. İlk olarak q 'a' noktasında olsun. γ bir eğri Γ üzerinde olsun. γ bir eği Γ üzerinde hareket edecek. q 'a' noktadan 'b' noktaya götürmek için ne kadar Enerji harcanmış,



Cevap



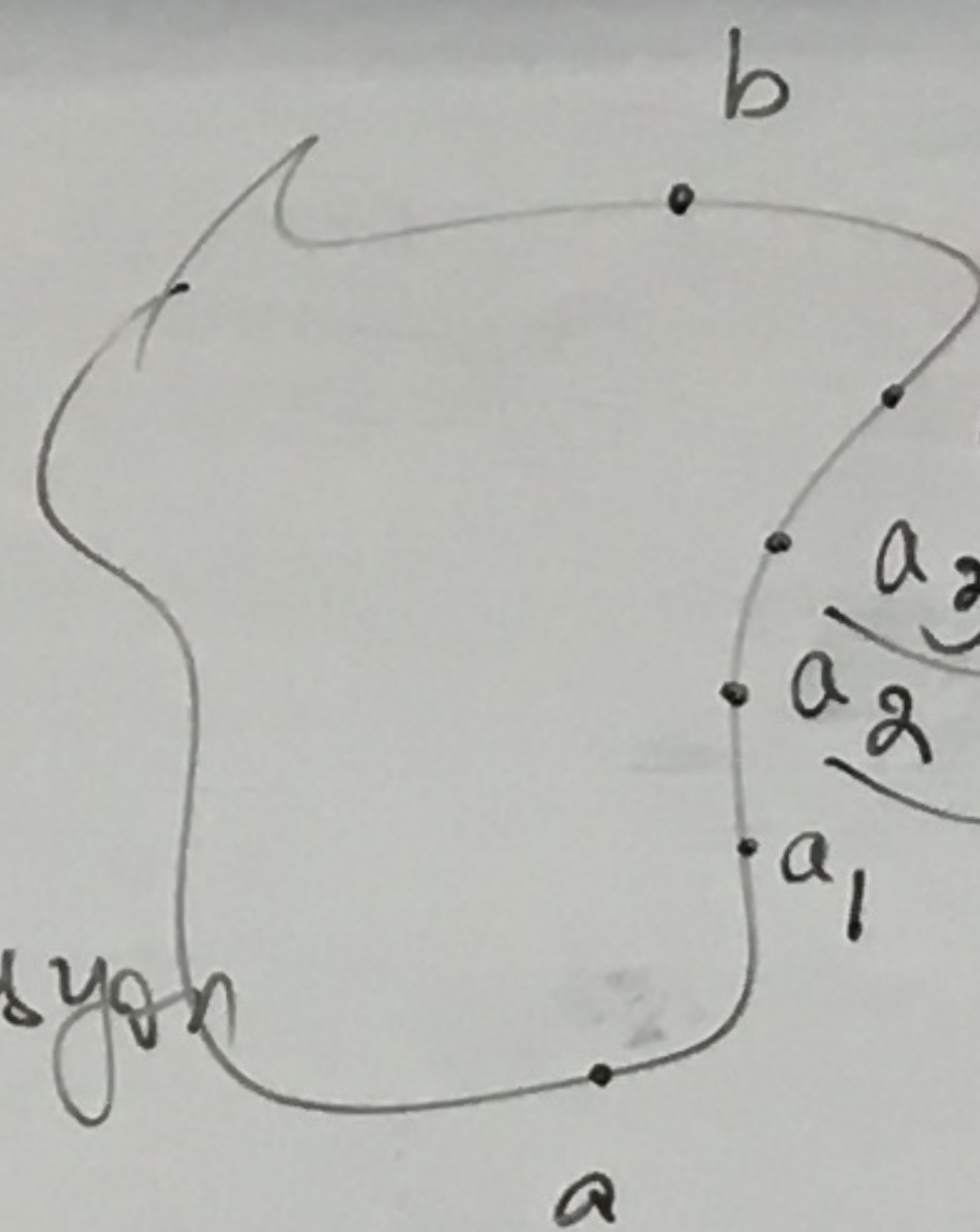
Newton Yasa.

Harcanan enerji

$F(a) \cdot w \cdot q.$

$\hookrightarrow F(a) \cdot (b-a) \cdot q$

Parametrizasyon



$F(a_2) \cdot (a_3 - a_2) \cdot q$

$F(a_1) \cdot (a_2 - a_1) \cdot q$

(Toplam Enerji : $\sum F(a_i) \cdot (a_{i+1} - a_i) \cdot q$)

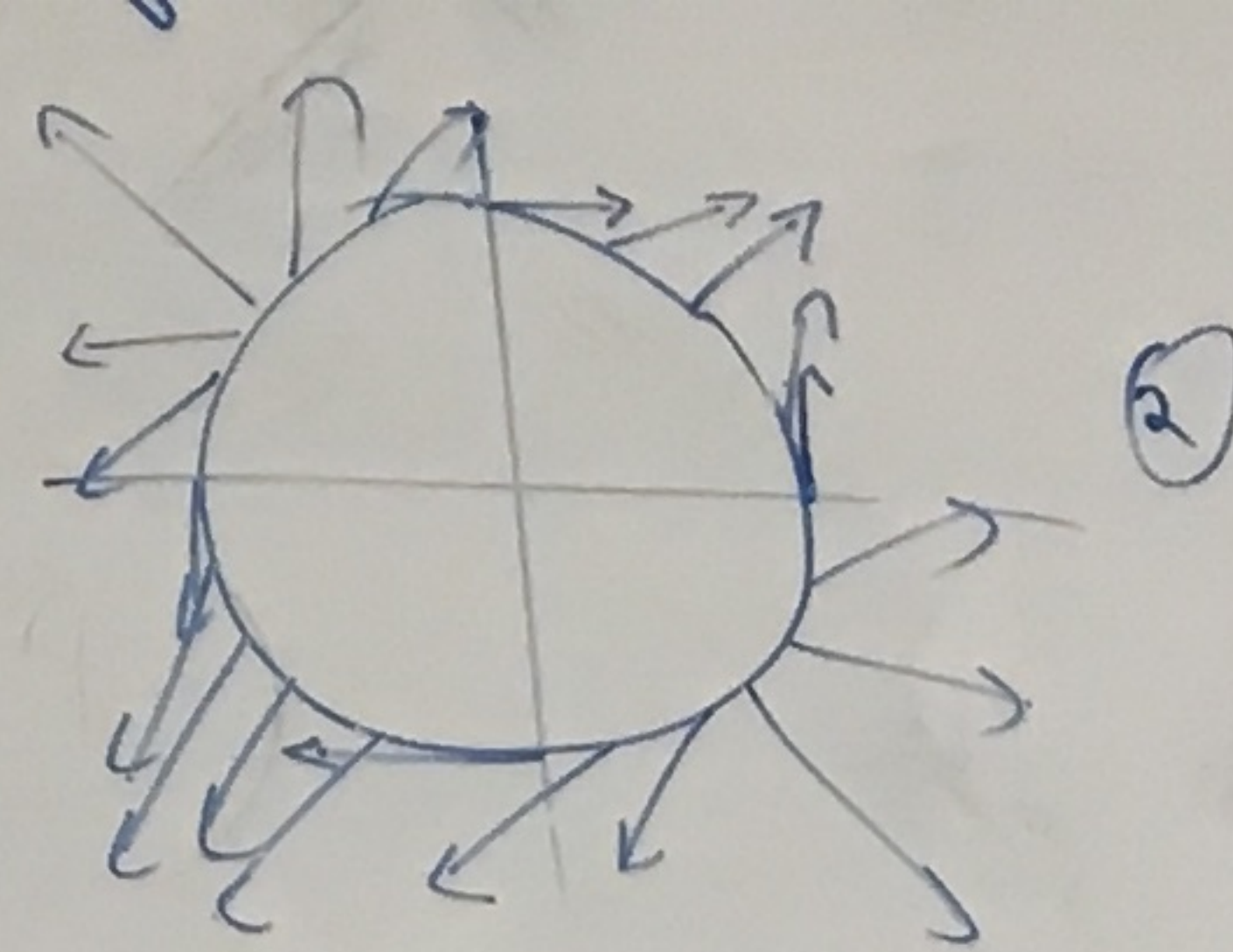
Tanım
2.2.2

$\Gamma = \{ \gamma(t) : 0 \leq t \leq T \} \subset \mathbb{R}^n$ bir eğri
 $\gamma : [0, T] \rightarrow \mathbb{R}^n$ olsun. $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$ bir e' vektör alan olsun.

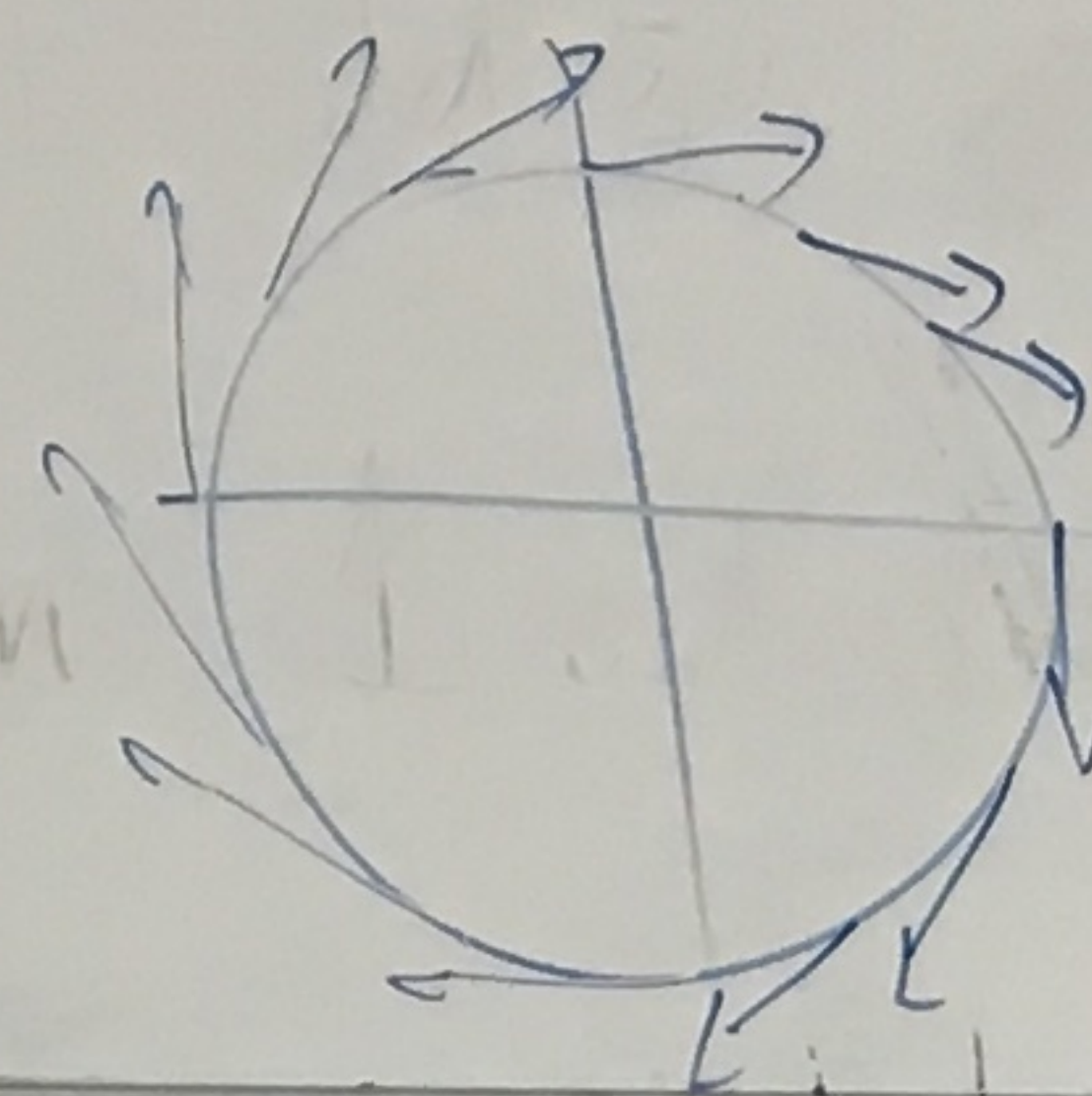
$$③ F(x, y) = \left(\frac{y}{x^2+y^2}, \frac{-x}{x^2+y^2} \right) = (\sin t, -\cos t)$$

S' üzerinde

$$F(\cos t, \sin t) = (\sin t, -\cos t)$$



$$F(\cos t, \sin t) = (\sin t, -\cos t)$$



0 zaman F' 'ın T üzerindeki
eğrisel integral

$$\int_T F \cdot dT = \int_0^{2\pi} F(\gamma(t)) \cdot \gamma'(t) dt$$

Örnek 2.8.3

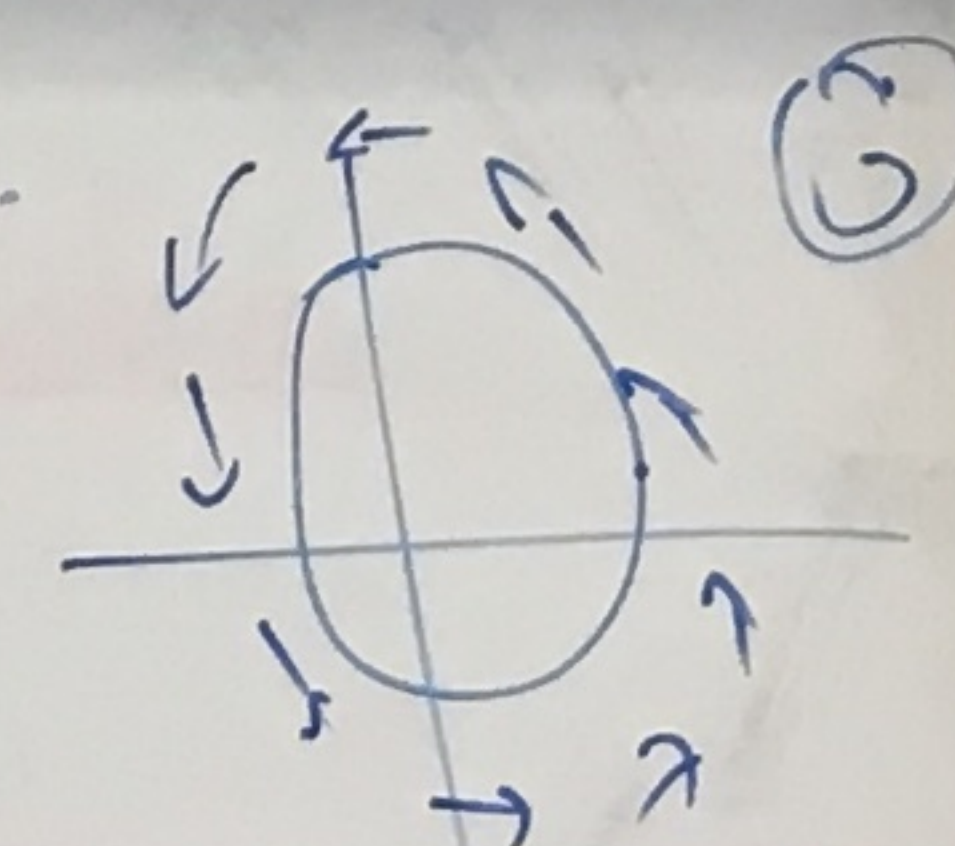
$$F(x, y) = \left(\frac{y}{x^2+y^2}, \frac{-x}{x^2+y^2} \right)$$

$$\Gamma = S'$$

$$\int_T F \cdot dT \text{ hesaplayalım}$$

$$T = \{ (\cos t, \sin t) : 0 \leq t \leq 2\pi \}$$

$$\gamma(t)$$



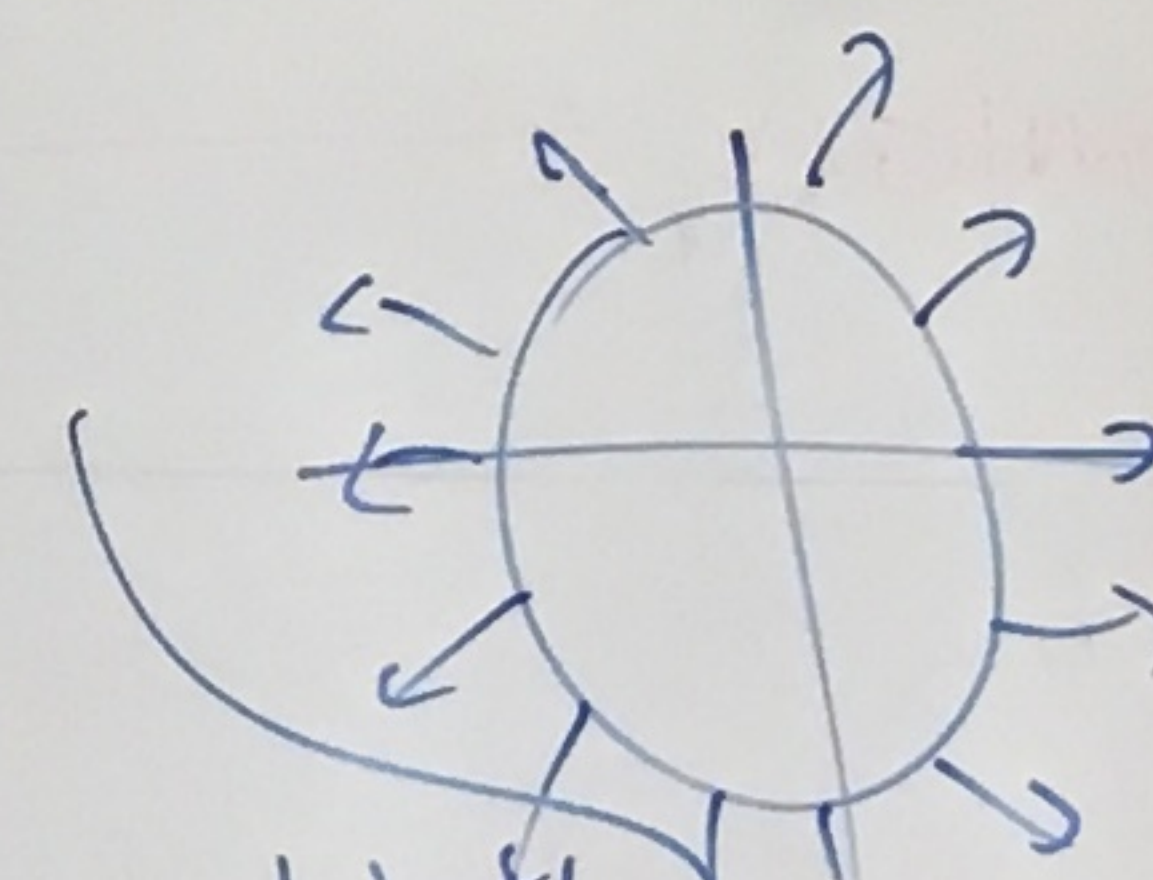
$$F(\gamma(t)) = F(\cos t, \sin t)$$

$$= (\sin t, -\cos t)$$

$$\gamma'(t) = (-\sin t, \cos t)$$

$$\int_T F \cdot dT = \int_0^{2\pi} (\sin t, -\cos t) \cdot (-\sin t, \cos t) dt$$

$$= \int_0^{2\pi} 1 - 1 dt = -2\pi$$



$$\left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right)$$

Örnek 2.8.3

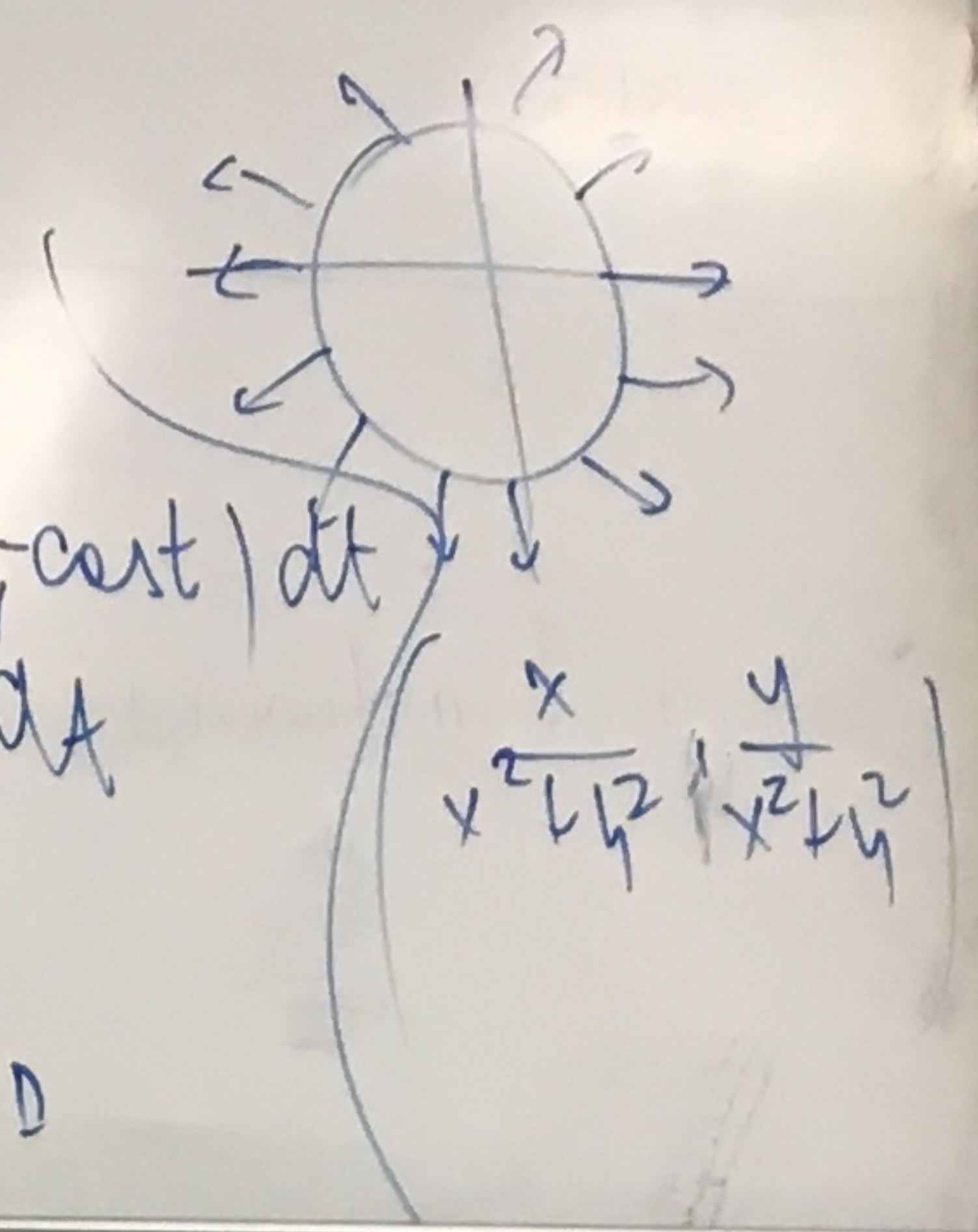
$$F(x, y) = \left(\frac{y}{x^2+y^2}, \frac{-x}{x^2+y^2} \right)$$

$\Gamma = S'$
 $\int_{\Gamma} F \cdot dT$ hesaplayınız.

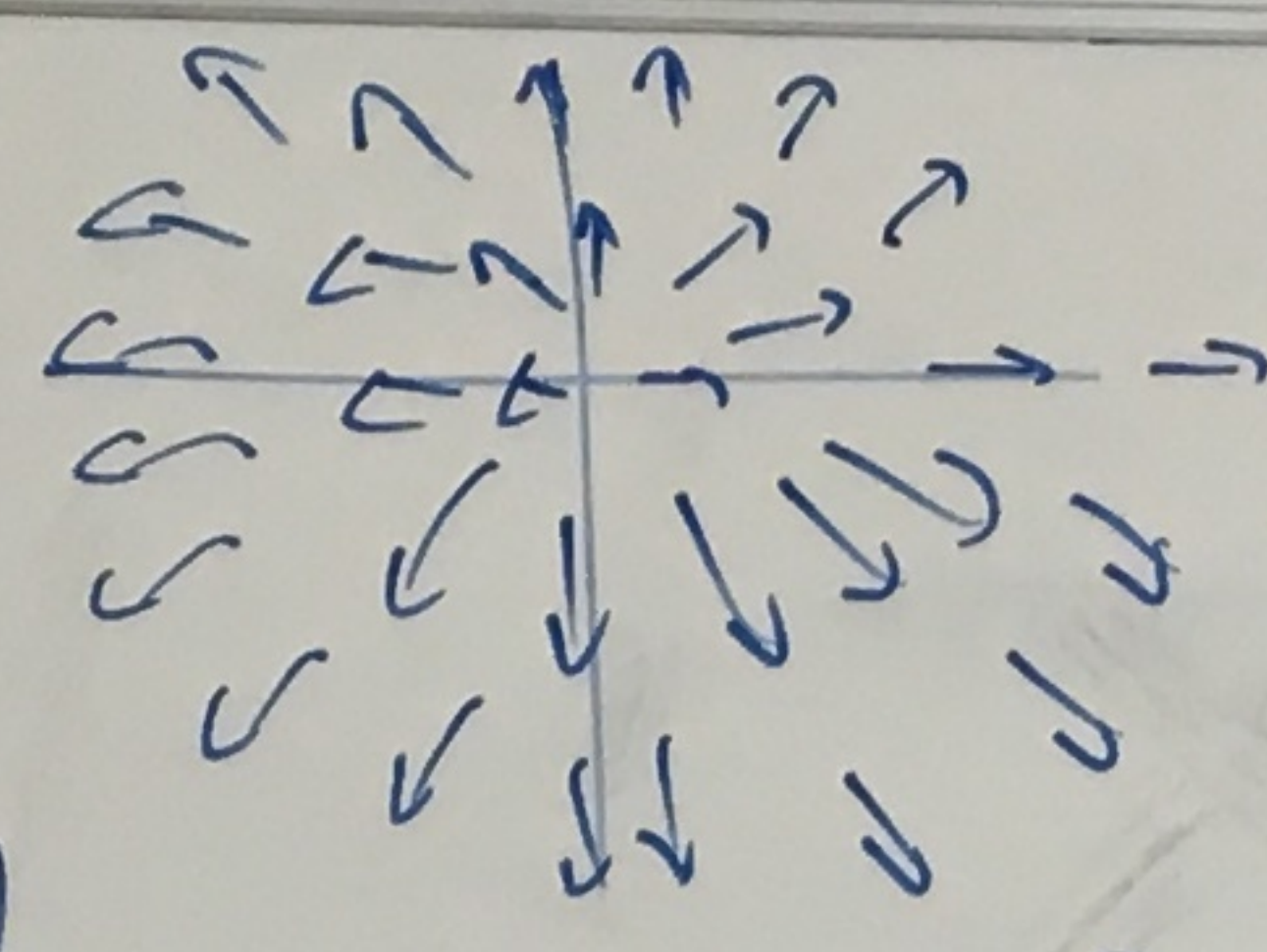
$$r'(t) = (-\sin t, \cos t)$$

$$\int_{\Gamma} F \cdot dT = \int_0^{2\pi} (\sin t, \cos t) \cdot (-\sin t, \cos t) dt$$

$$= \int_0^{2\pi} (-1) dt = -2\pi$$

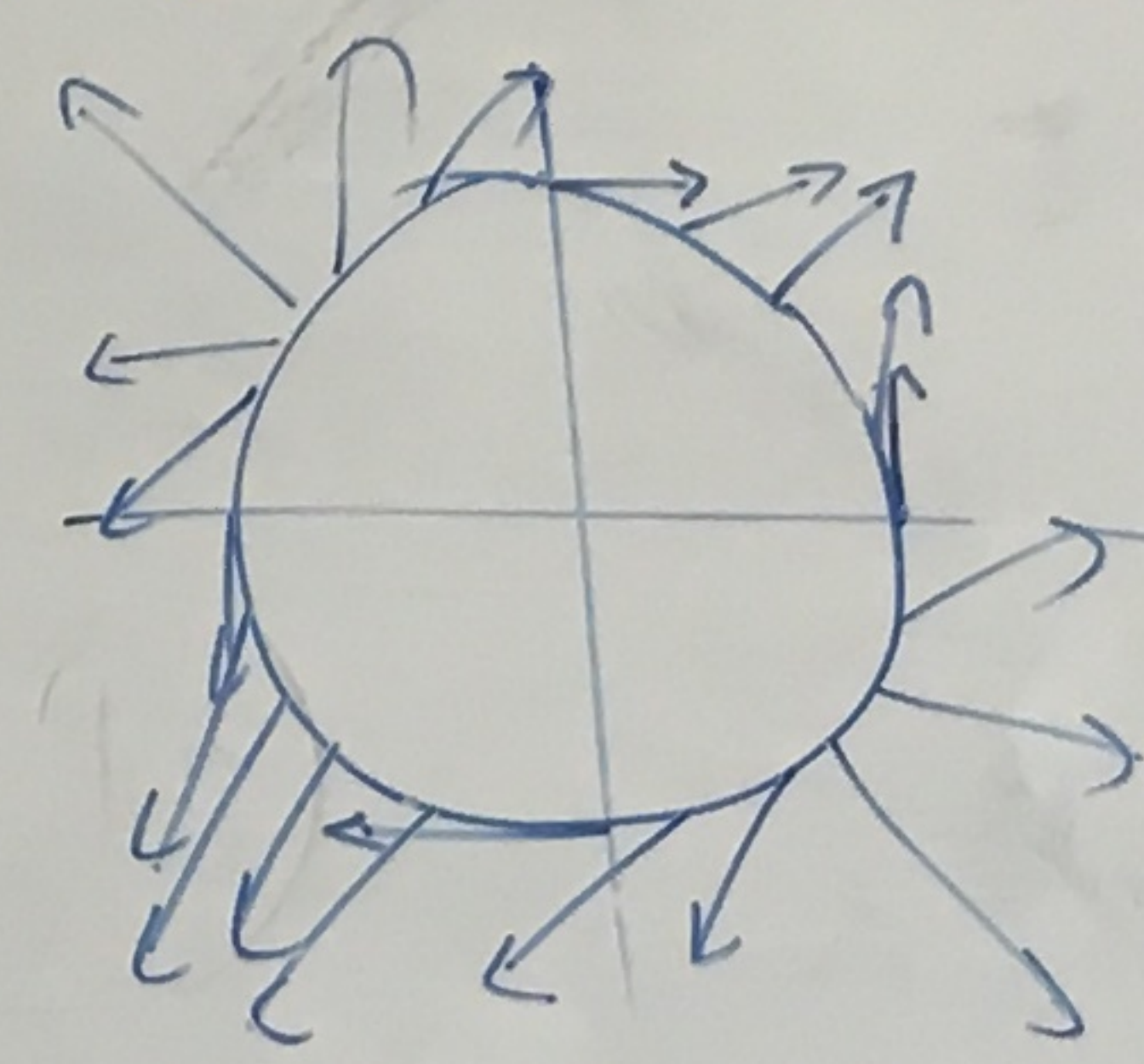


① $F(x, y) = \left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right) = (\cos t, \sin t)$



①

② $F(x, y) = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right) = (-\sin t, \cos t)$

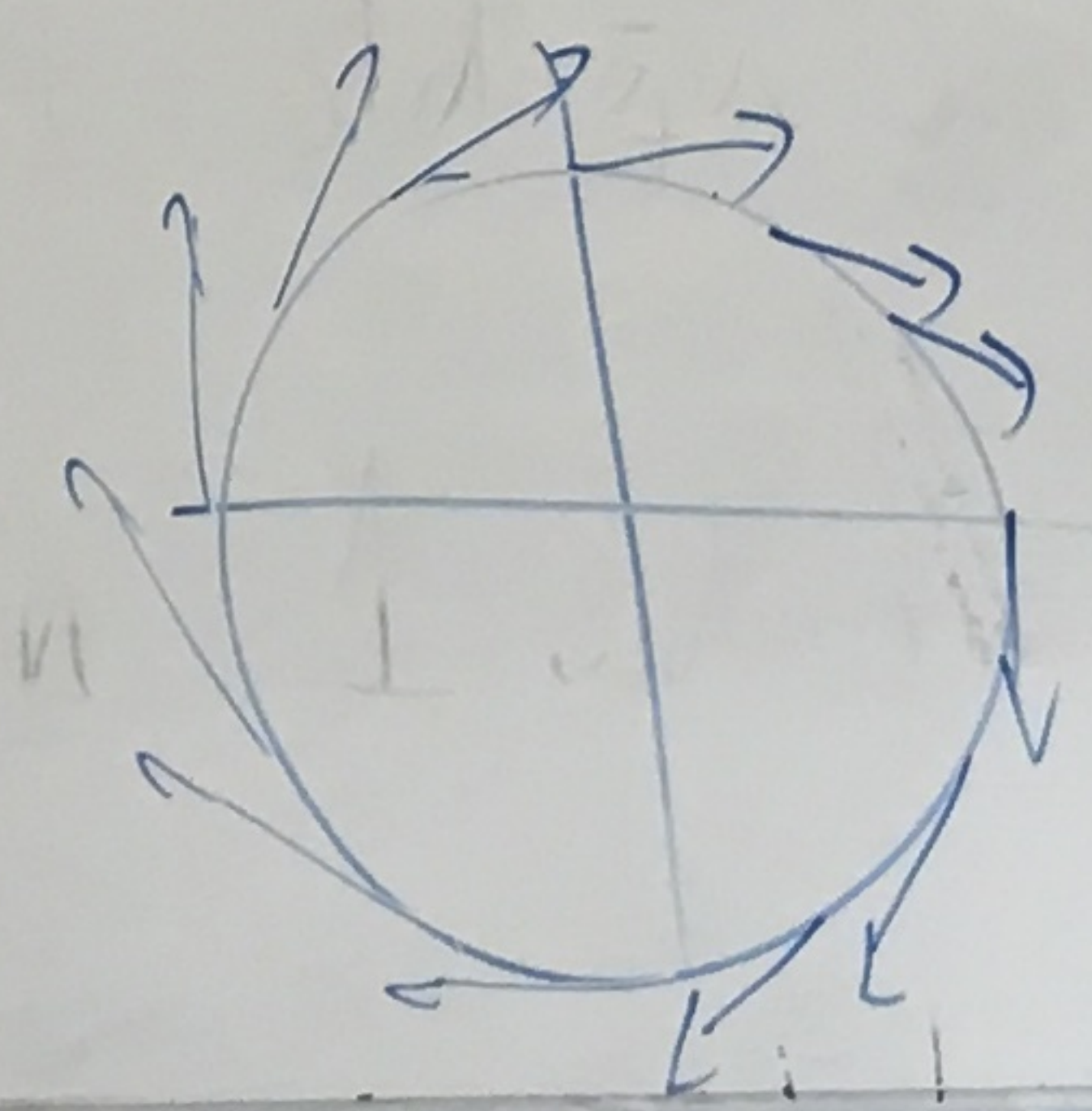


②

③ $F(x, y) = \left(\frac{y}{x^2+y^2}, \frac{-x}{x^2+y^2} \right) = (\sin t, -\cos t)$

S' üzerinde

$$F(\cos t, \sin t) = (\sin t, -\cos t)$$



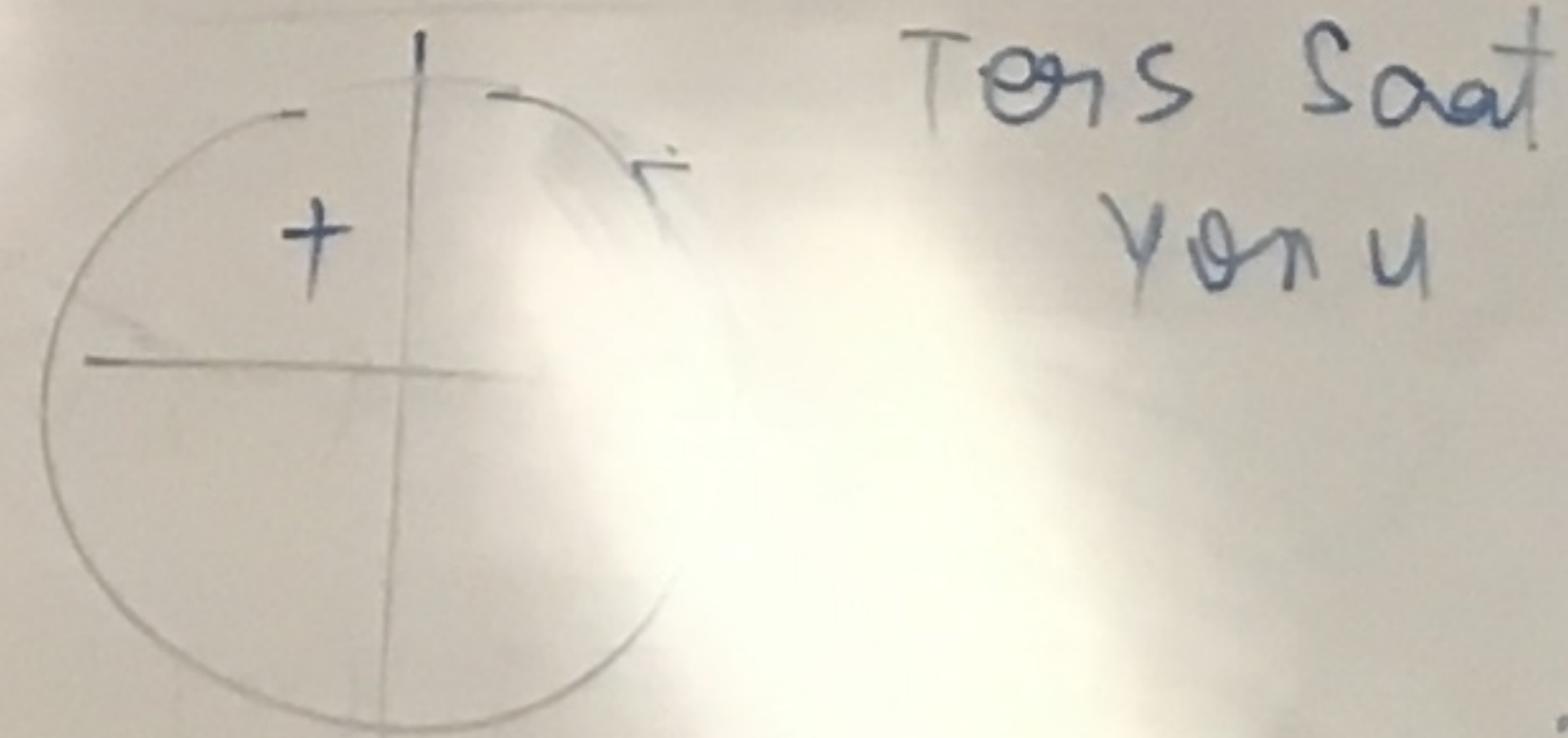
③

$r(\cos t, \sin t) = (\sin t, \cos t)$
 $r = 1$ dsim-
 $F(\cos t, \sin t) = (\sin t, \cos t)$

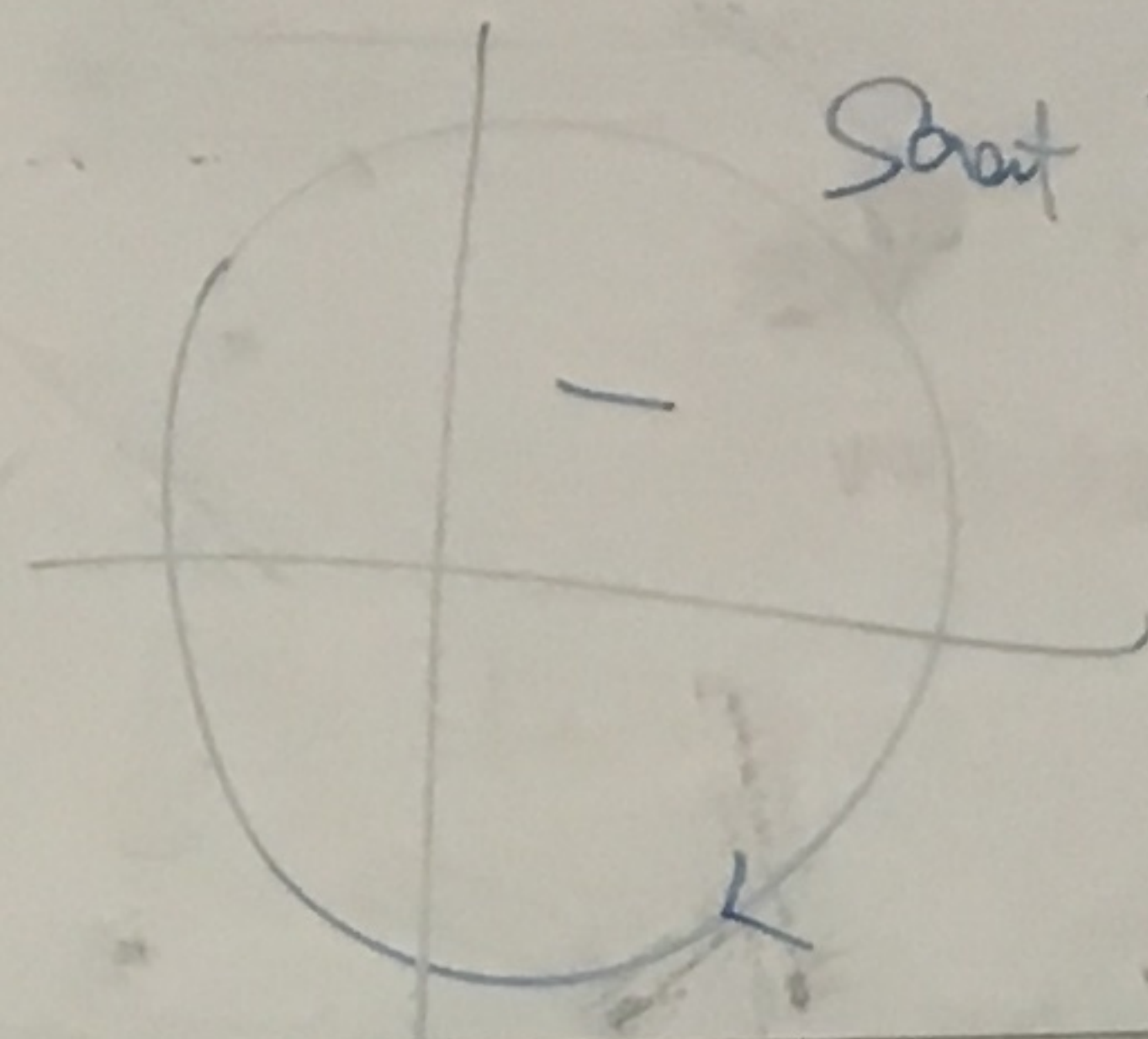
④

Bir orientasyon dir.

$$S = \{ (\cos t, \sin t) : 0 \leq t \leq 2\pi \}$$



$$S' = \{ (\cos t, -\sin t) : 0 \leq t \leq 2\pi \}$$



Γ hesaplanırken Γ' 'nin orientasyon belirlemeliyiz. (Ters orientasyon ise, Integral (-1) ile)

Saat yönü değişecek)

2.2.4 Orientasyon

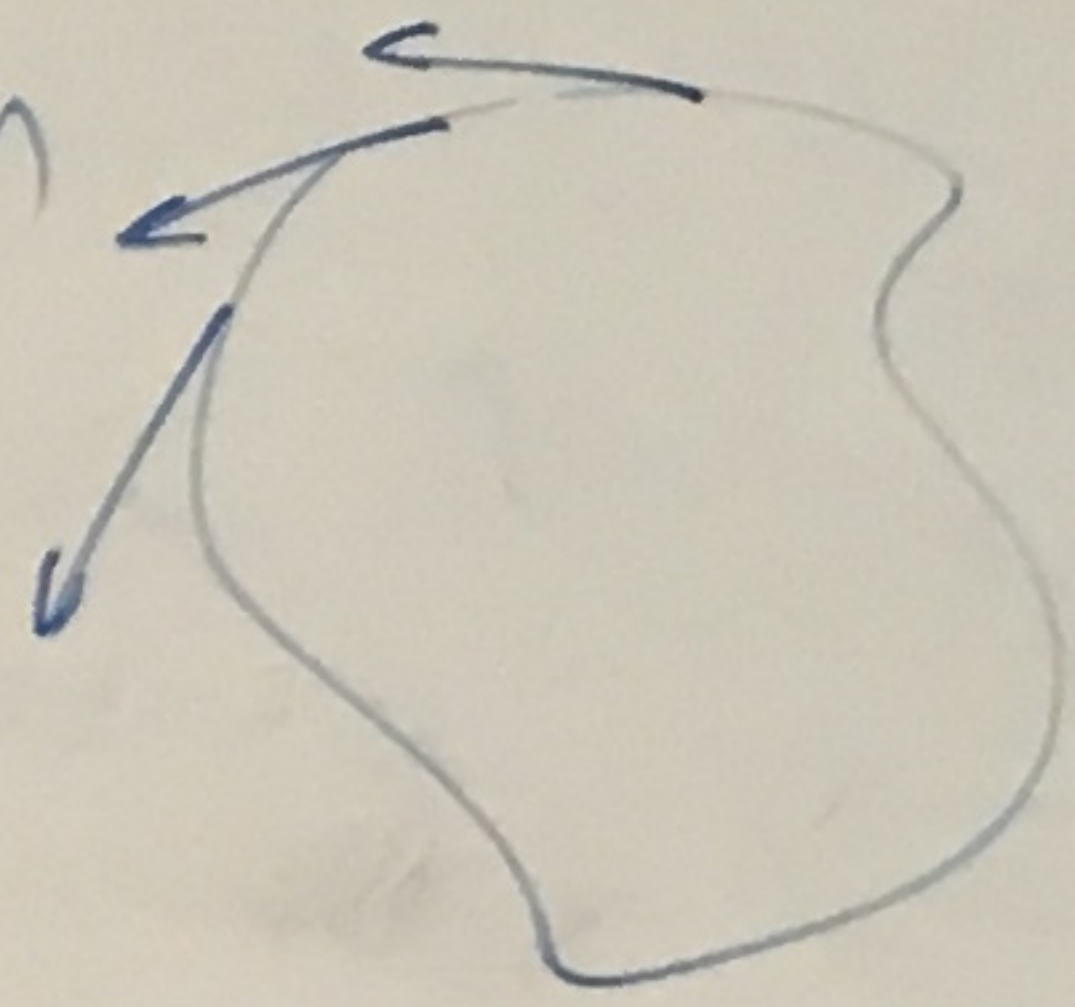
$\Gamma \subset \mathbb{R}^2$ bir eğri olsun (e').

Sürekliliği şekli her nokta

$A \in \Gamma$ için uzunluk

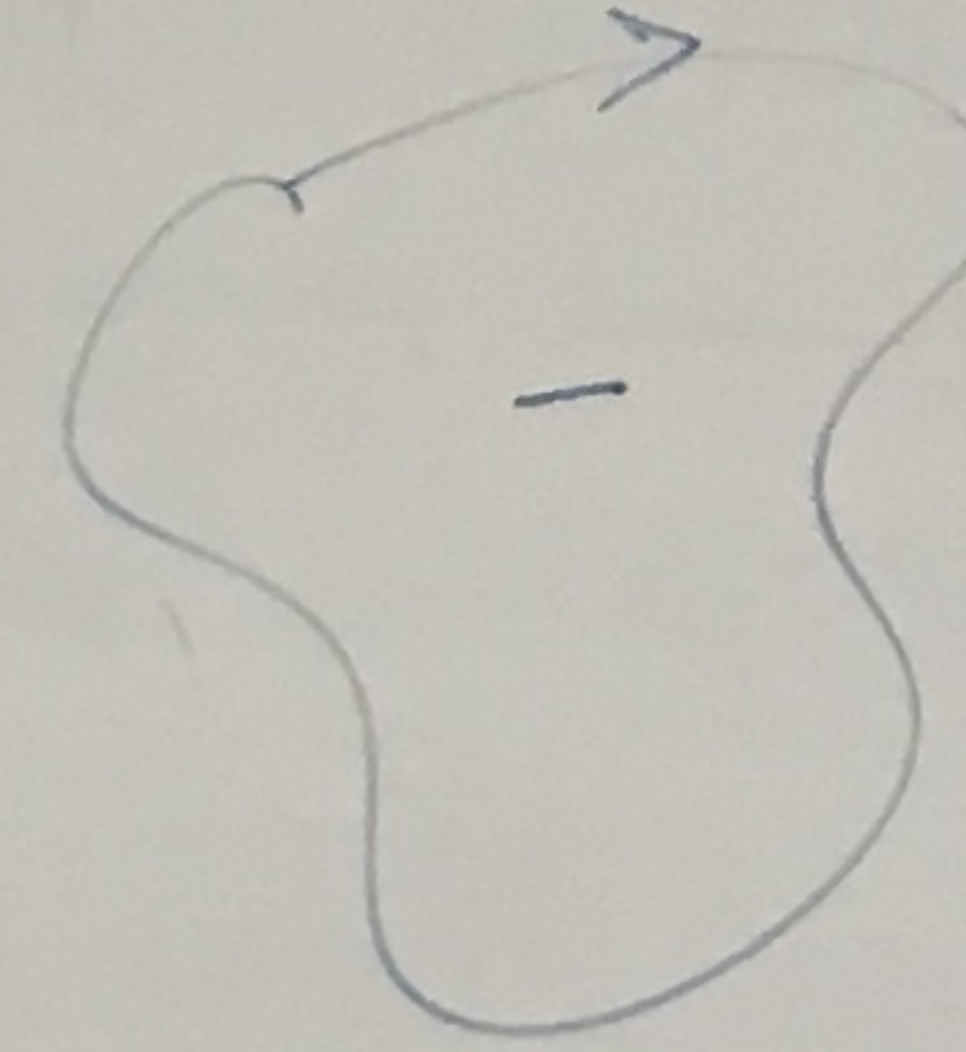
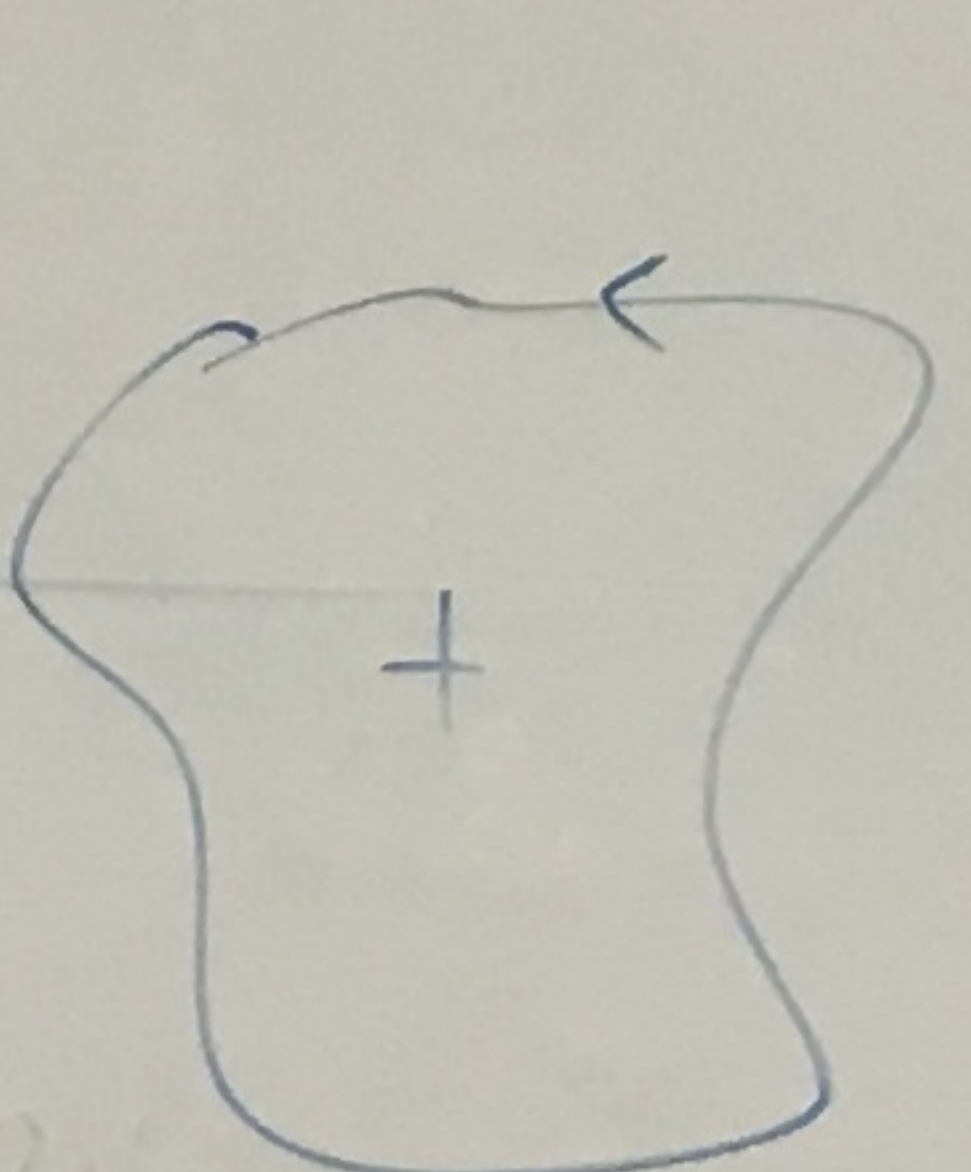
1 olan teğet vektör seçelim.

Bu bir orientasyon denir.



Not Bu eğri üzerinde hareket etmek için bir yön verin.

Eğer $\Gamma \subset \mathbb{R}^2$ içinde ise,



orientasyona göre kapalı alan hep sol da ise, bu + ve

orientasyon denir. (Yoksa -ve orientasyon).

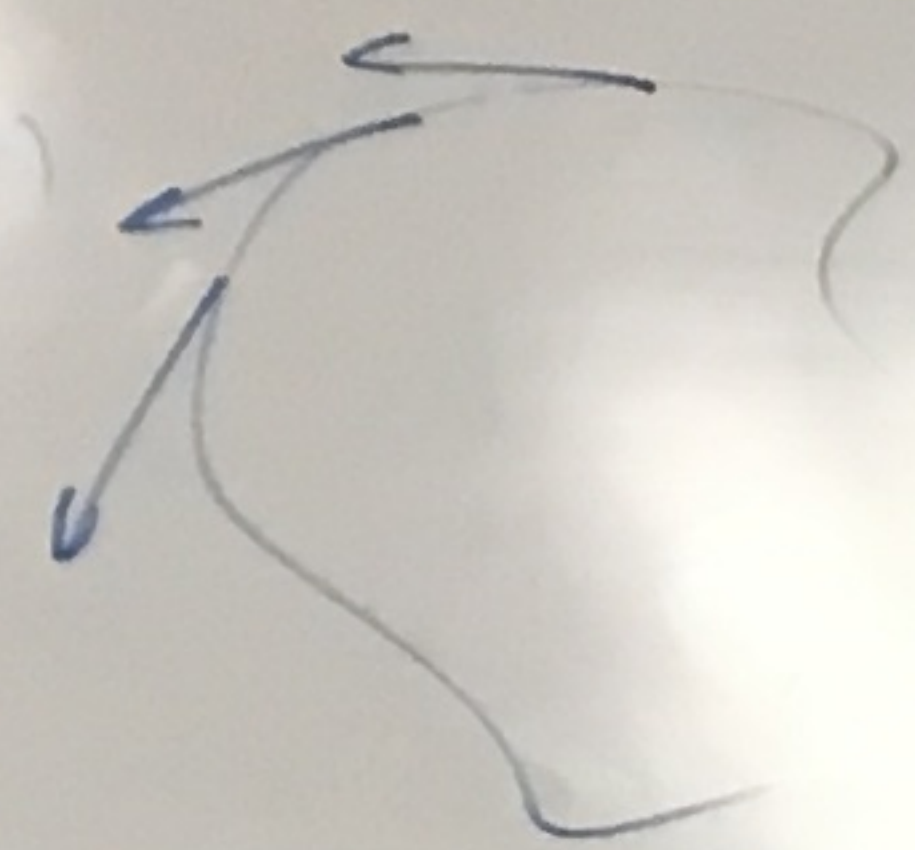
Not $\Gamma = \{ \gamma(t) : 0 \leq t \leq T \}$

Eğer hiçbir noktada

$\gamma'(t) \neq 0$, bu Düzgün parametrisasyon denir.

$\rightarrow e'$ parametrisasyon.

Alan teğet vektör seçelim.
Bu bir parametrisasyon
denir.



Not Bu eğri üzerinde hareket etmek
için bir yön verin.

hep sağ da ise, bu +ve
orientasyon denir. (Yoksa -ve
orientasyon).

$$\text{Not } \Gamma = \{ \gamma(t) : 0 \leq t \leq T \}$$

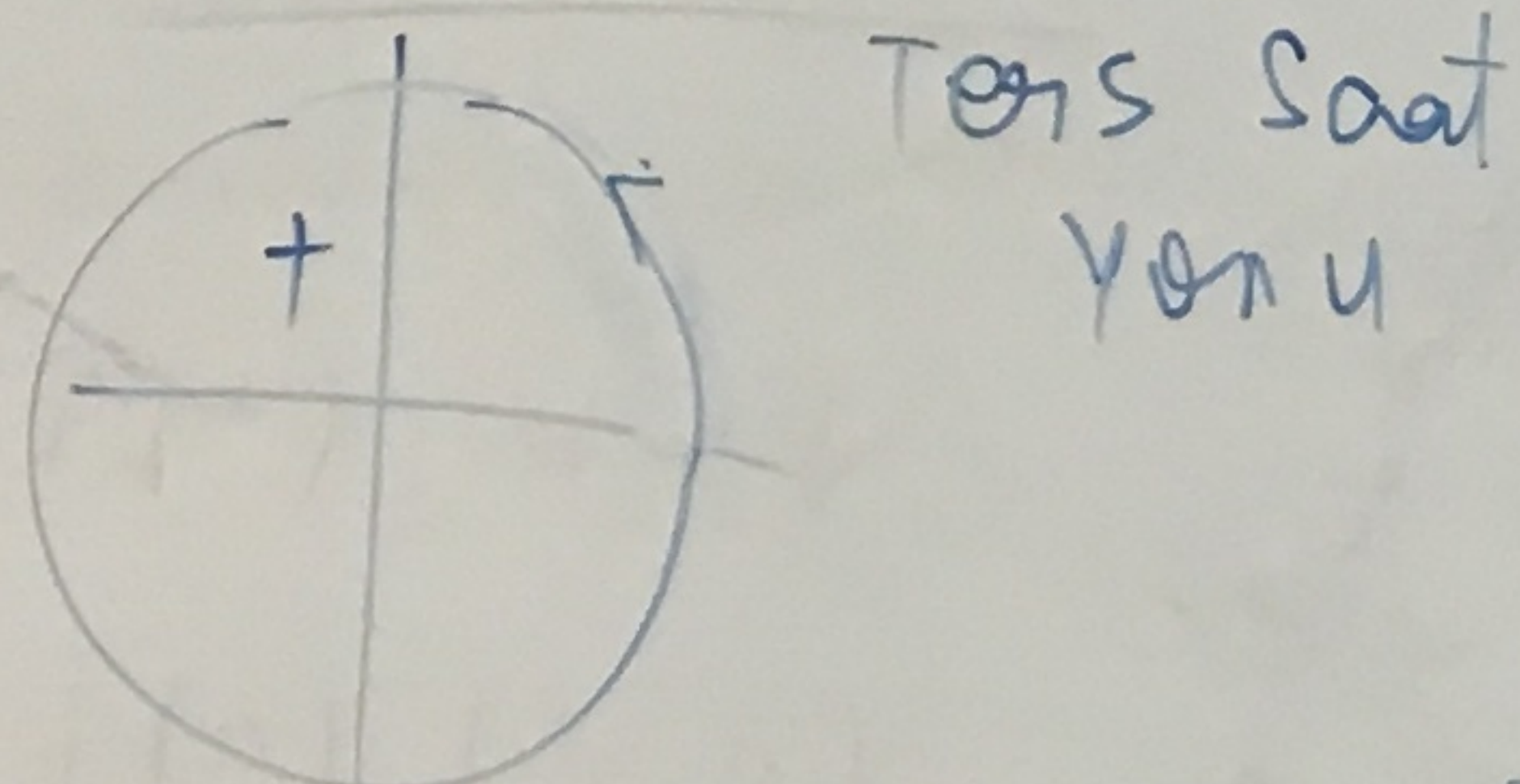
Eğer hiçbir noktada $\gamma'(t) \neq 0$, bu Düzgün parametrisasyon denir.

$\rightarrow e'$ parametrisasyon.

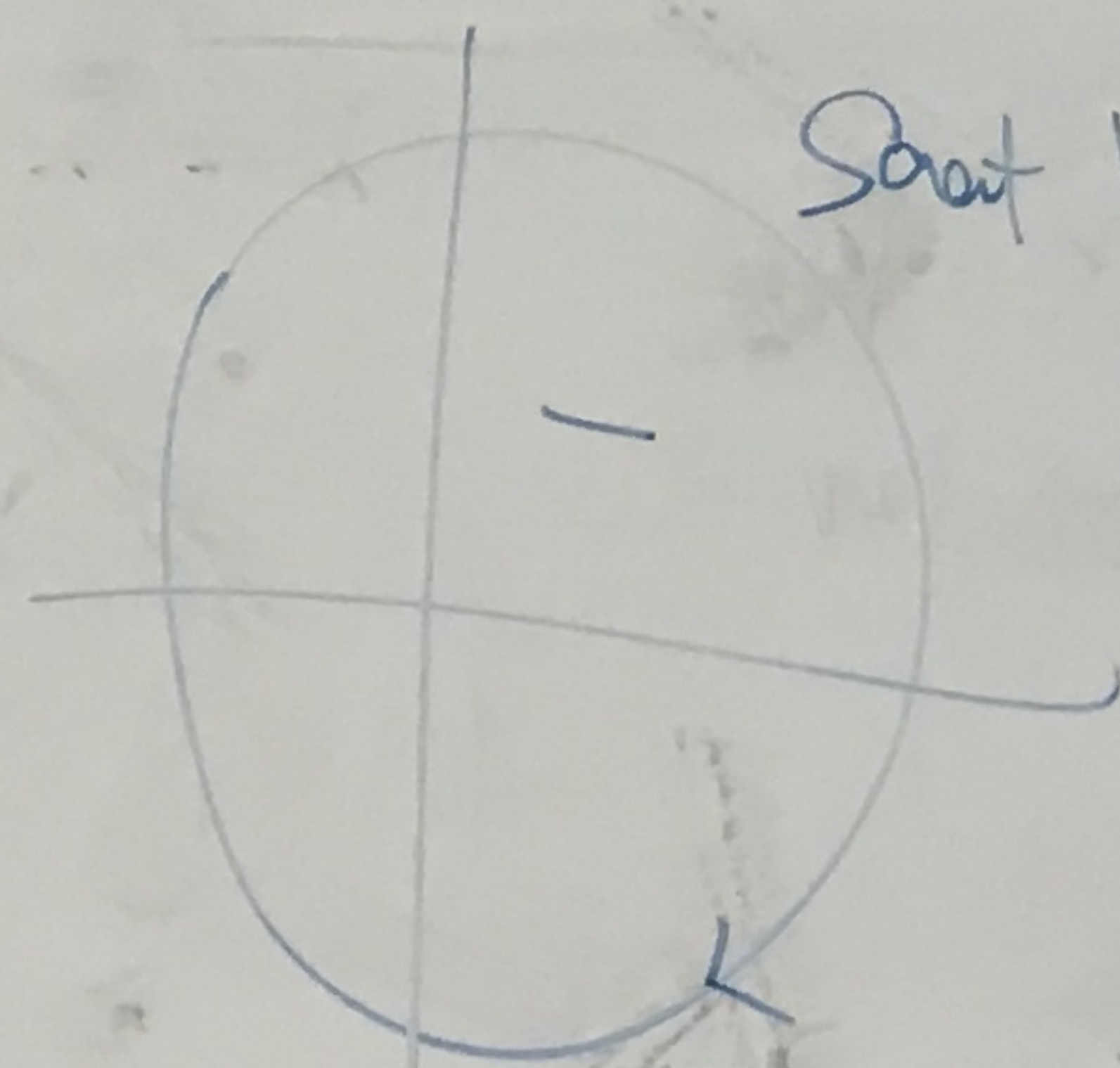
$$t \rightarrow \frac{\gamma'(t)}{\|\gamma'(t)\|}$$

Bir orientasyon dir.

$$\rightarrow S' : \{ (\cos t, \sin t) : 0 \leq t \leq 2\pi \}$$



$$S' = \{ (\cos t, -\sin t) : 0 \leq t \leq 2\pi \}$$



Eğrisel Integral

$$\int_{\Gamma} F \cdot d\mathbf{T}$$

hesaplanırken Γ' 'nin orientasyon
belirlenmelidir. (Ters orientasyon
ise, Integral (-1) ile
Saat yönü değişecek).

$$\Gamma_2: \begin{aligned} \gamma(t) &= (1, t) \\ \gamma'(t) &= (0, 1) \\ t(\gamma(t)) &= (1-t, 1) \end{aligned}$$

$$\int_{\Gamma_3} t \cdot dT$$

$$\begin{aligned} \gamma(t) &= (1-t, 0) \\ \gamma'(t) &= (-1, 0) \\ F(\gamma(t)) &= (0, (1-t)^2) \end{aligned}$$

$$\int_0^1 (0, (1-t)^2) \cdot (-1, 0) dt = 0$$

$$\frac{1}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{1}{3}$$

örnek 2.2.5

$$\Gamma = \left\{ \begin{aligned} &(0,0), (1,0), (1,1) \\ &\text{köşeler olan} \\ &\text{üçgen olsun} \end{aligned} \right\}$$

orientasyon -ve olsun.
negatif

$$F = (xy, x^2) \text{ olsun.}$$

$$\begin{aligned} &\hookrightarrow xy \hat{i} + x^2 \hat{j} \\ &\int_{\Gamma} F \cdot dT \text{ hesaplayınız.} \end{aligned}$$

$$\left\{ (t, t) : 0 \leq t \leq 1 \right\} \xrightarrow{\Gamma_1} \left\{ (1, 1-t) : 0 \leq t \leq 1 \right\} \xrightarrow{\Gamma_2} \left\{ (1-t, 0) : 0 \leq t \leq 1 \right\}$$

$$\int_{\Gamma_1} F \cdot dT =$$

$$\begin{aligned} &= \int_0^1 F(\gamma(t)) \cdot \gamma'(t) dt \\ &= \int_0^1 (t^2, t^2) \cdot (1, 1) dt \\ &= \int_0^1 2t^2 dt = \frac{2}{3} \end{aligned}$$

$$\gamma(t) = (t, t)$$

$$\gamma'(t) = (1, 1)$$

$$F(\gamma(t)) = (t^2, t^2)$$

7

üçgen olsun
 oryantasyon -ve olsun.
 negatif

$$F = (xy, x^2) \text{ olsun.}$$

$$= xy \hat{i} + x^2 \hat{j}$$

$\int F \cdot dT$ hesaplayalım.

$$\int_{\Gamma_1} F \cdot dT =$$

$$\int F(\gamma(t)) \cdot \gamma'(t) dt$$

$$= \int_0^1 (t^2, t^2) \cdot (1, 1) dt$$

$$= \int_0^1 (t^2 + t^2) dt = 2 \int_0^1 t^2 dt = \frac{2}{3}$$

$$\gamma(t) = (t, t)$$

$$\gamma'(t) = (1, 1)$$

$$F(\gamma(t)) = (t^2, t^2)$$

$$\rightarrow \{(1-t, 0) : 0 \leq t \leq 1\}$$

$$\int_{\Gamma_2} F \cdot dT = \int_0^1 (1-t, 1) \cdot (0, -1) dt = \int_0^1 -1 dt = -1.$$

$$\Gamma_2: \gamma(t) = (1, 1-t)$$

$$\gamma'(t) = (0, -1)$$

$$F(\gamma(t)) = (1-t, 1)$$

$$\int_{\Gamma} = \int_{\Gamma_1} + \int_{\Gamma_2} + \int_{\Gamma_3} = \frac{2}{3} - 1 + 0 = -\frac{1}{3}$$

$$\int_{\Gamma_3} F \cdot dT$$

$$\gamma(t) = (1-t, 0)$$

$$\gamma'(t) = (-1, 0)$$

$$F(\gamma(t)) = (0, (1-t)^2)$$

$$\int_0^1 (0, (1-t)^2) \cdot (-1, 0) dt = 0$$

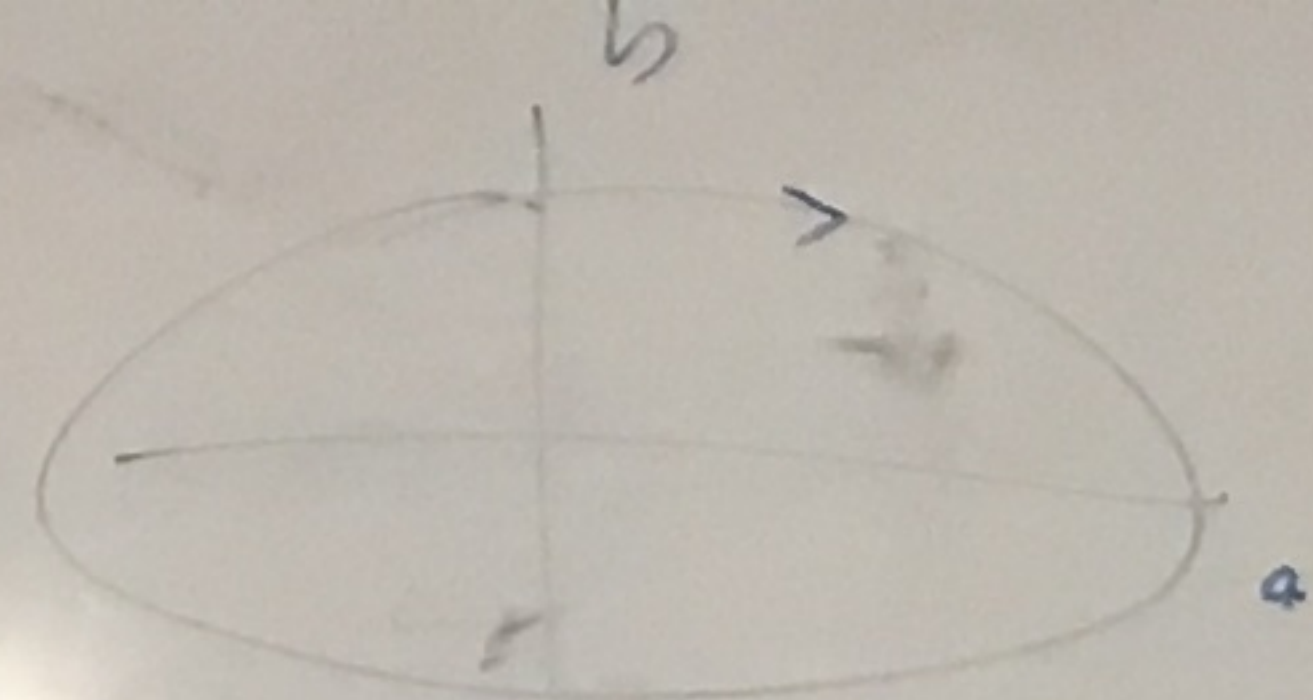
$$F(x,y) = (xy, x^2y)$$

$$\int_C F \cdot dT \text{ hesaplayınız.}$$

$$r(t) = (a \cos t, -b \sin t) \quad 0 \leq t \leq 2\pi$$

$$r'(t) = (-a \sin t, -b \cos t)$$

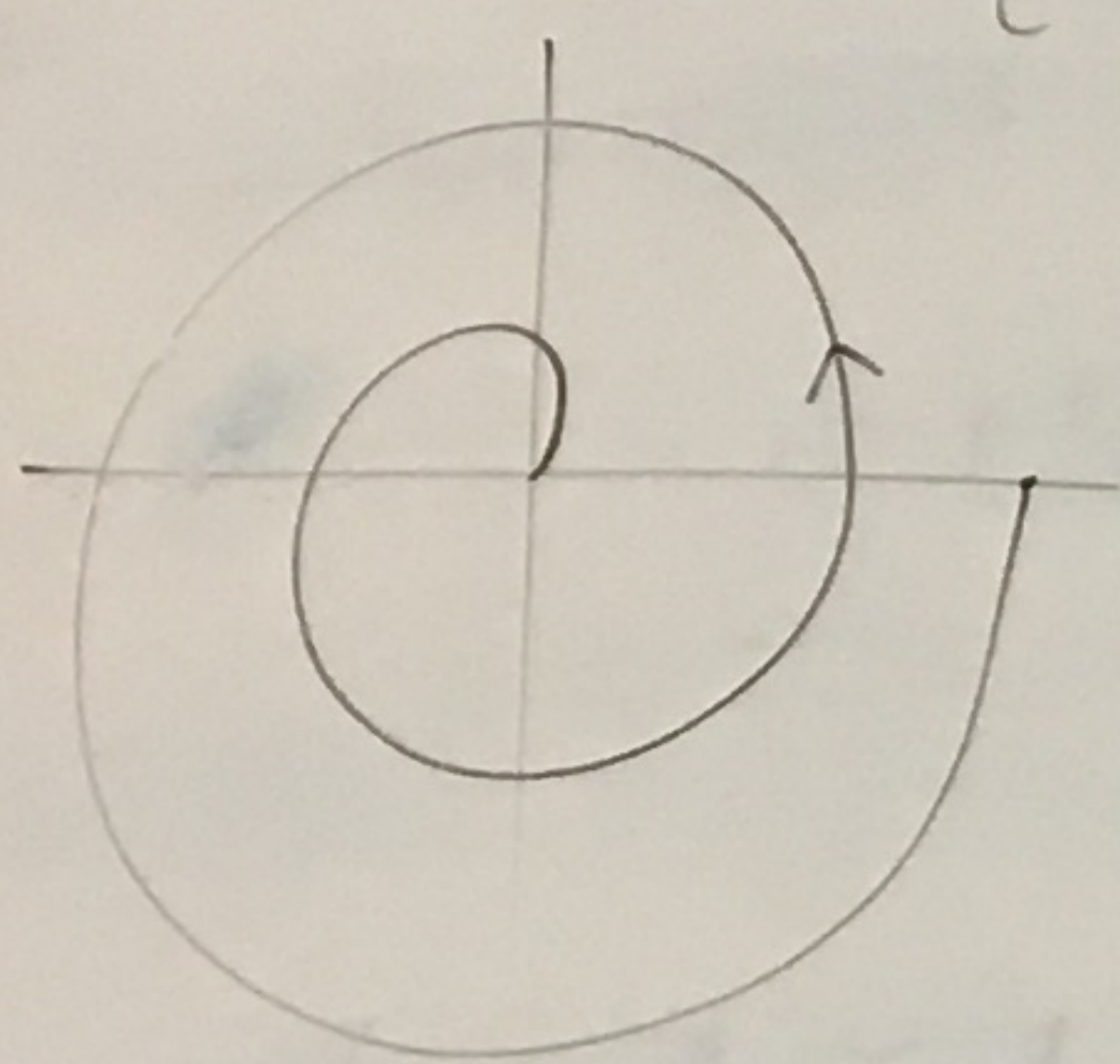
$$F(r(t)) = (-ab \sin t \cos t, a^2 b^2 \sin^2 t \cos^2 t)$$



$$\begin{aligned} &= \int_0^{2\pi} (-ab \sin t \cos t, a^2 b^2 \sin^2 t \cos^2 t) \cdot (-a \sin t, -b \cos t) dt \\ &= \int_0^{2\pi} (ab^2 \sin^2 t \cos^2 t - a^3 b \sin^2 t \cos^3 t) dt \\ &= 0 \end{aligned}$$

$$\int_C F \cdot dT = \int_0^T F(r(t)) \cdot r'(t) dt$$

Örnek 2.2.6 $\Gamma = \{ r = \theta \mid 0 \leq \theta \leq 4\pi \}$ (+m orientasyon)
spiral.



$$F(r, \theta) = (r\theta, \theta) \quad \left(\begin{array}{l} r \text{ dan} \\ \text{Ko-ordinat} \end{array} \right)$$

$$\int_C F \cdot T \text{ hesaplayınız.}$$

$$r(t) = t \quad r'(t) = (\cos t - t \sin t, \sin t + t \cos t)$$

$$\int_C F \cdot dT$$

$$= \int_0^{4\pi} (t^2 \cos t, t^2 \sin t) \cdot (\cos t - t \sin t, \sin t + t \cos t) dt$$

örnek $\Gamma = \{ (t \cos t, t \sin t) : 0 \leq t \leq 4\pi \}$

Cartesian

$$F(r \cos \theta, r \sin \theta) = (r\theta \cos \theta, r\theta \sin \theta)$$

$$F(t \cos t, t \sin t) = (t^2 \cos t, t^2 \sin t)$$

$$\begin{aligned} &= \int_0^{4\pi} (t^2 \cos^2 t - t^3 \cos t \sin t + t^2 \sin^2 t + t^3 \sin t \cos t) dt \\ &= \int_0^{4\pi} t^2 dt = \frac{(4\pi)^3}{3} \end{aligned}$$

$$\gamma(t) = (t \cos t, t \sin t) : 0 \leq t \leq 4\pi \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Cartesian}$$

$$F(r \cos \theta, r \sin \theta) = (r \cos \theta, r \sin \theta)$$

$$F(t \cos t, t \sin t) = (t^2 \cos t, t^2 \sin t)$$

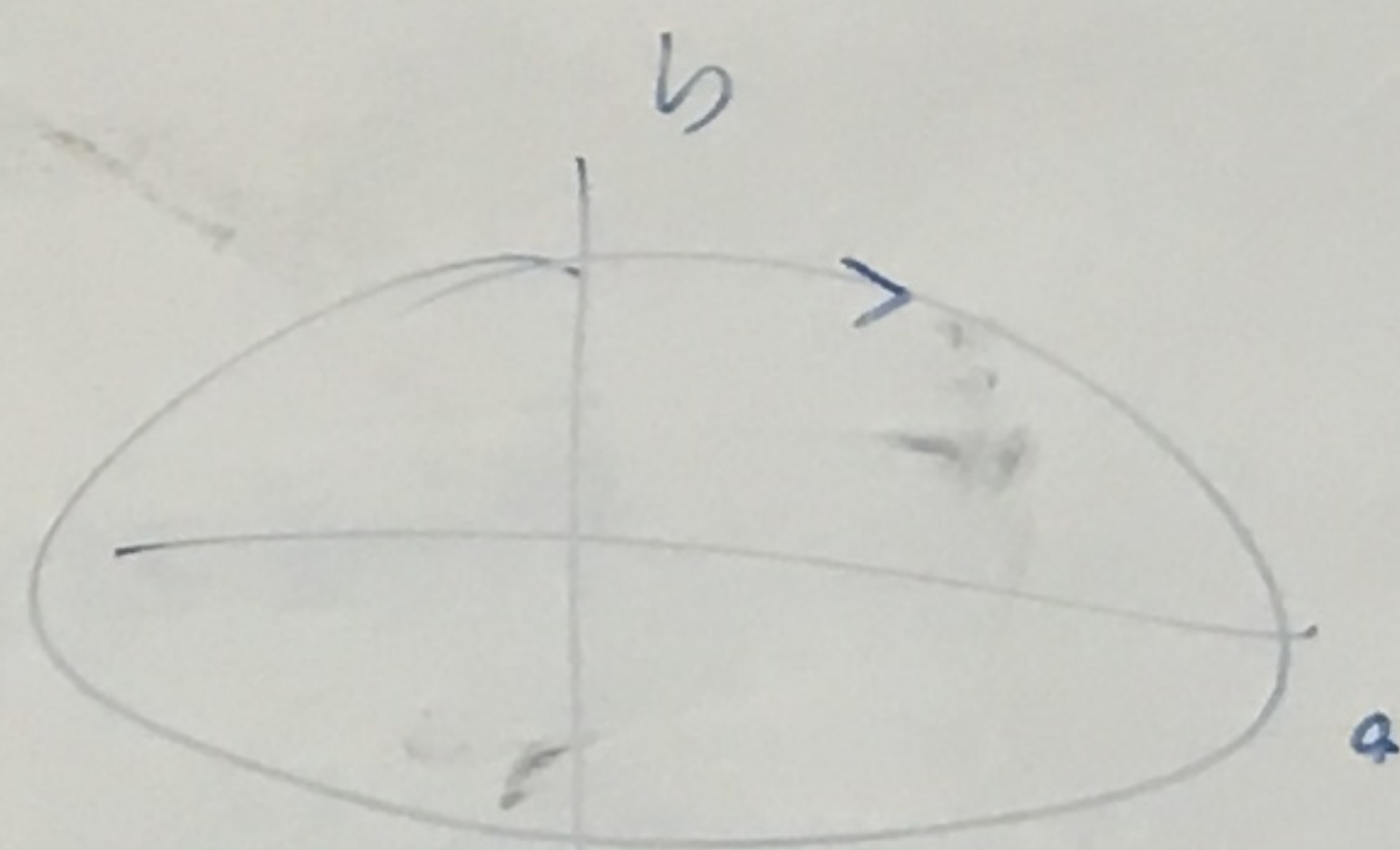
$$\begin{aligned} &= \int_0^{4\pi} (t^2 \cos^2 t - t^3 \cos t \sin t + t^2 \sin^2 t + t^3 \sin t \cos t) dt \\ &= \int_0^{4\pi} t^2 dt = \frac{(4\pi)^3}{3} \end{aligned}$$

örnek 2.2.7

$$T = \left\{ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \right\} \quad \text{—ve} \quad \text{orientasyon.}$$

$$F(x, y) = (xy, x^2 y)$$

$$\int_T F \cdot dT \quad \text{hesaplayalım.}$$



$$\gamma(t) = (a \cos t, -b \sin t) \quad 0 \leq t \leq 2\pi$$

$$\gamma'(t) = (-a \sin t, -b \cos t)$$

$$F(\gamma(t)) = (-ab \sin t \cos t, a^2 b^2 \sin^2 t \cos^2 t)$$

$$\begin{aligned} &\int_T F \cdot dT \\ &= \int_0^{2\pi} \begin{pmatrix} -ab \sin t \cos t, a^2 b^2 \sin^2 t \cos^2 t \end{pmatrix} \cdot \begin{pmatrix} -a \sin t, -b \cos t \end{pmatrix} dt \end{aligned}$$

$$\begin{aligned} &= \int_0^{2\pi} (ab \sin^2 t \cos t - a^2 b^3 \sin^2 t \cos^3 t) dt \\ &= 0 \end{aligned}$$

$$\begin{aligned} \Gamma &= \{ \gamma(t) : 0 \leq t \leq T \} \\ \int_T F \cdot dT &:= \int_0^T F(\gamma(t)) \gamma'(t) dt \end{aligned}$$

(10)