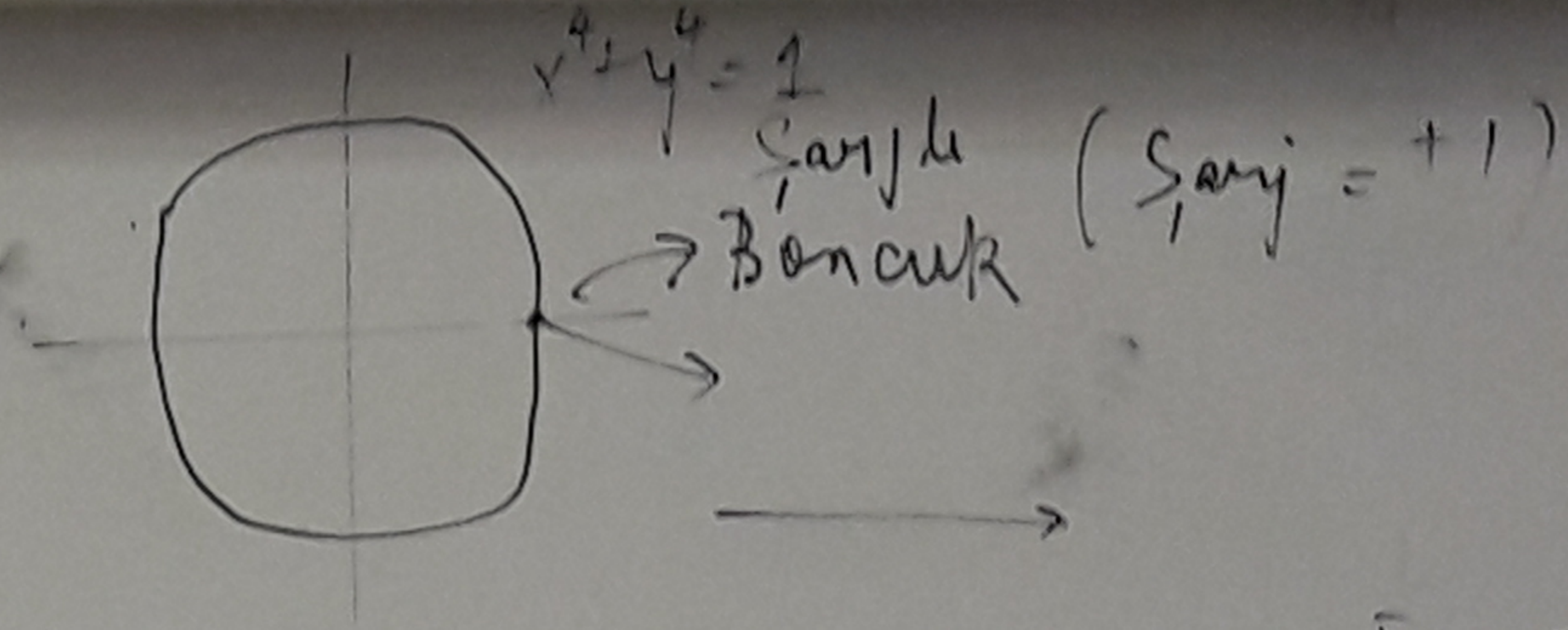


Soru 2



Elektrik alan:

$$F = ma$$

→ ivme

$$E(x, y) = (x + 2y, 3x + 4y)$$

Bes, Şarjı sonra böncük nerede?

Vektör fonksiyonu eğri üzerinde

İntegre etmek lazım.

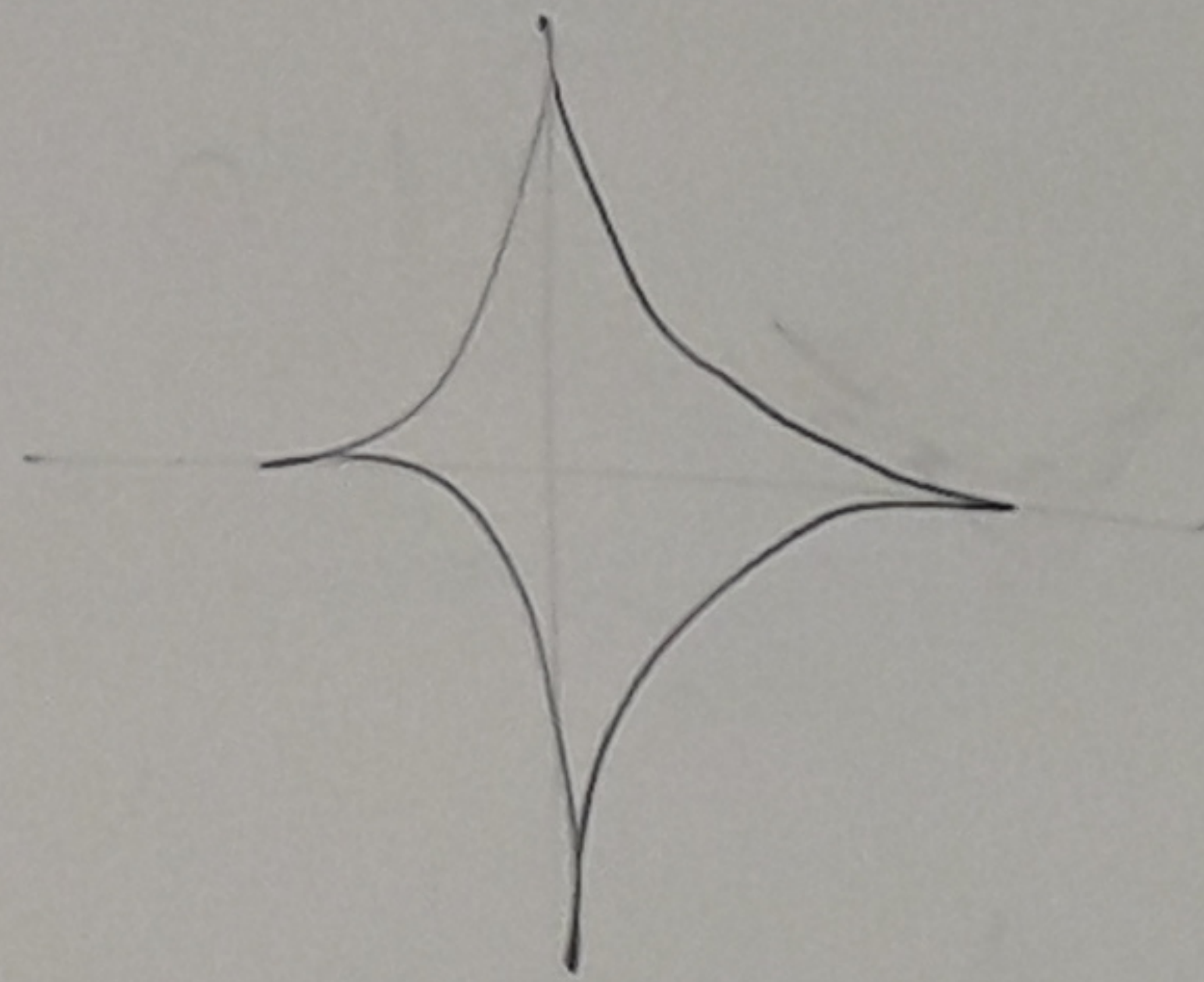
Vektör fonksiyonu eğri üzerinde
İntegre etmek lazım.

Soru 3

$$S: \left\{ (x, y) \mid x^{2/3} + y^{2/3} = 1 \right\}$$

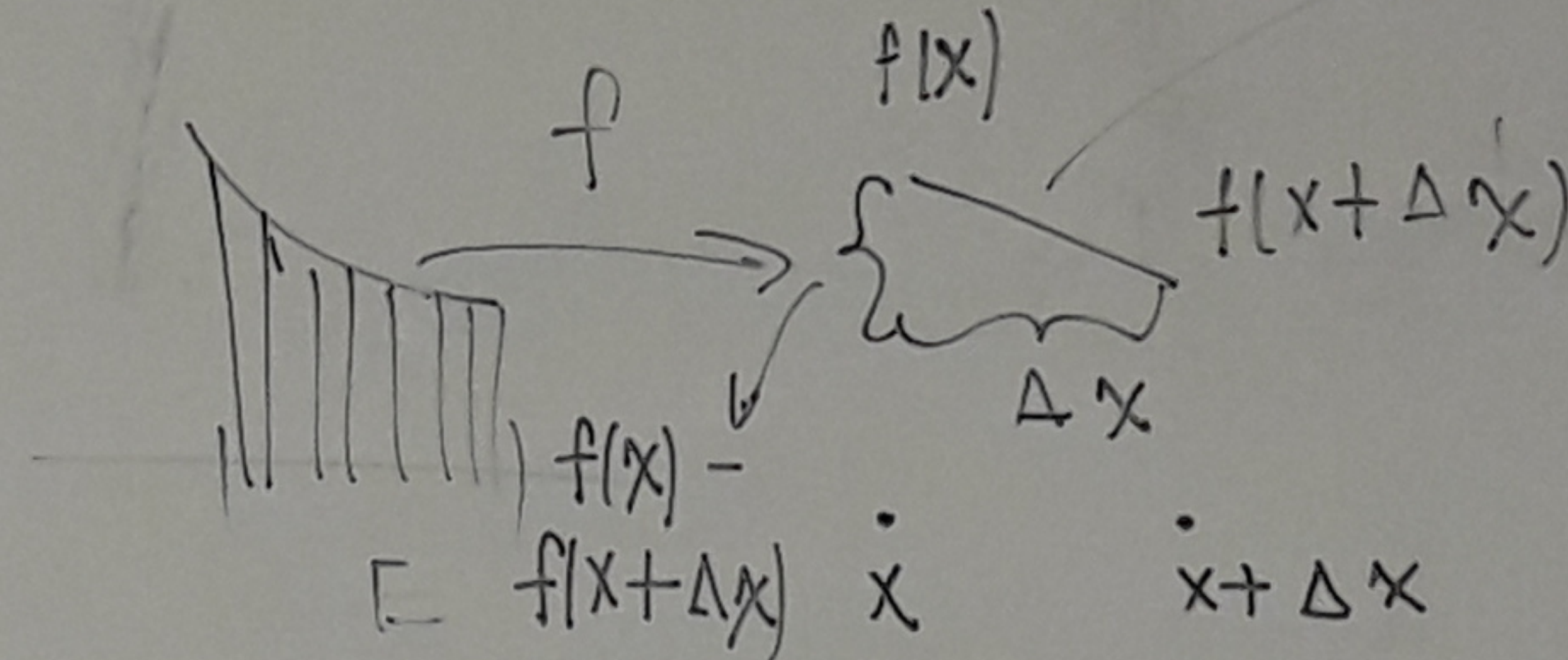
Astroid.

Bunun uzunluk n kadardır?



Tanım 2.1.1 $E \subset \mathbb{R}$ aralık.

$f: E \rightarrow \mathbb{R}$ e' olsun.



$$\sqrt{(\Delta x)^2 + [f(x+\Delta x) - f(x)]^2}$$

(2)

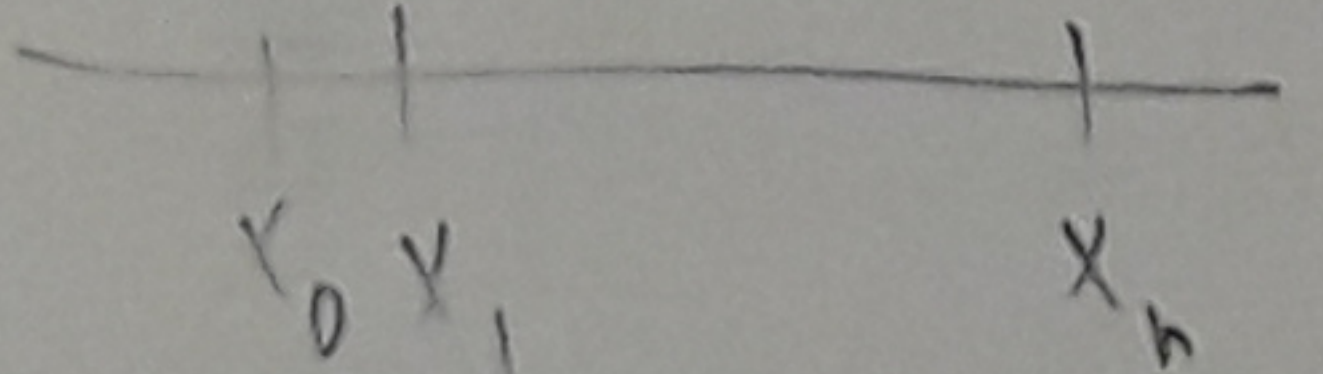
$$\psi(x) = (x, f(x))$$

$$\psi'(x) = (1, f'(x))$$

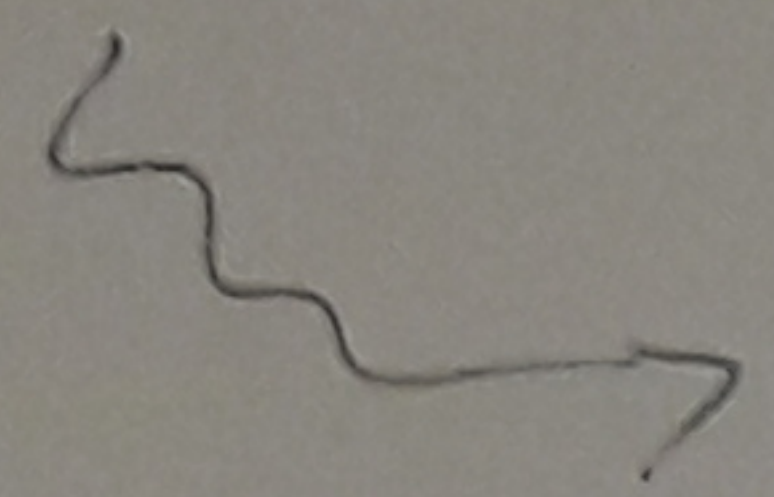
$$\|\psi'(x)\| = \sqrt{1 + f'(x)^2}$$

$$S: \{(x, f(x)) : x \in E\} \quad \text{Uzunluk?}$$

$$\text{uzunluk} \approx \sum_{i=1}^n \sqrt{[f(x_{i+1}) - f(x_i)]^2 + (x_{i+1} - x_i)^2}$$



$$x = \frac{\tan^2 \theta}{4}$$



b) Not: Aynı eğri için birden fazla parametrisasyon olabilir.

$$S = \{ (\cos^2 \theta, \sin^2 \theta) : \theta \in [0, \sqrt{2}\pi] \}$$

Ortalama Değer Teorisi

$$f(x_{i+1}) - f(x_i) = f'(c_i)(x_{i+1} - x_i)$$

$$c_i \in (x_i, x_{i+1})$$

uzunluk $\Rightarrow \sum \sqrt{1 + f'(c_i)^2} (x_{i+1} - x_i)$

$$x_{i+1} - x_i \rightarrow 0 \text{ ise,}$$

uzunluk $\rightarrow \int \sqrt{1 + f'(x)^2} dx$

Tanım 1-2-1

$E \subset \mathbb{R}$ aralık $f: E \rightarrow \mathbb{R}$ e'
 0 zaman, $S := \{ (x, f(x)) : x \in E \}$ 'nin VAY
 UZUNLUĞU

$$L(S) = \int_E \sqrt{1 + f'(x)^2} dx$$

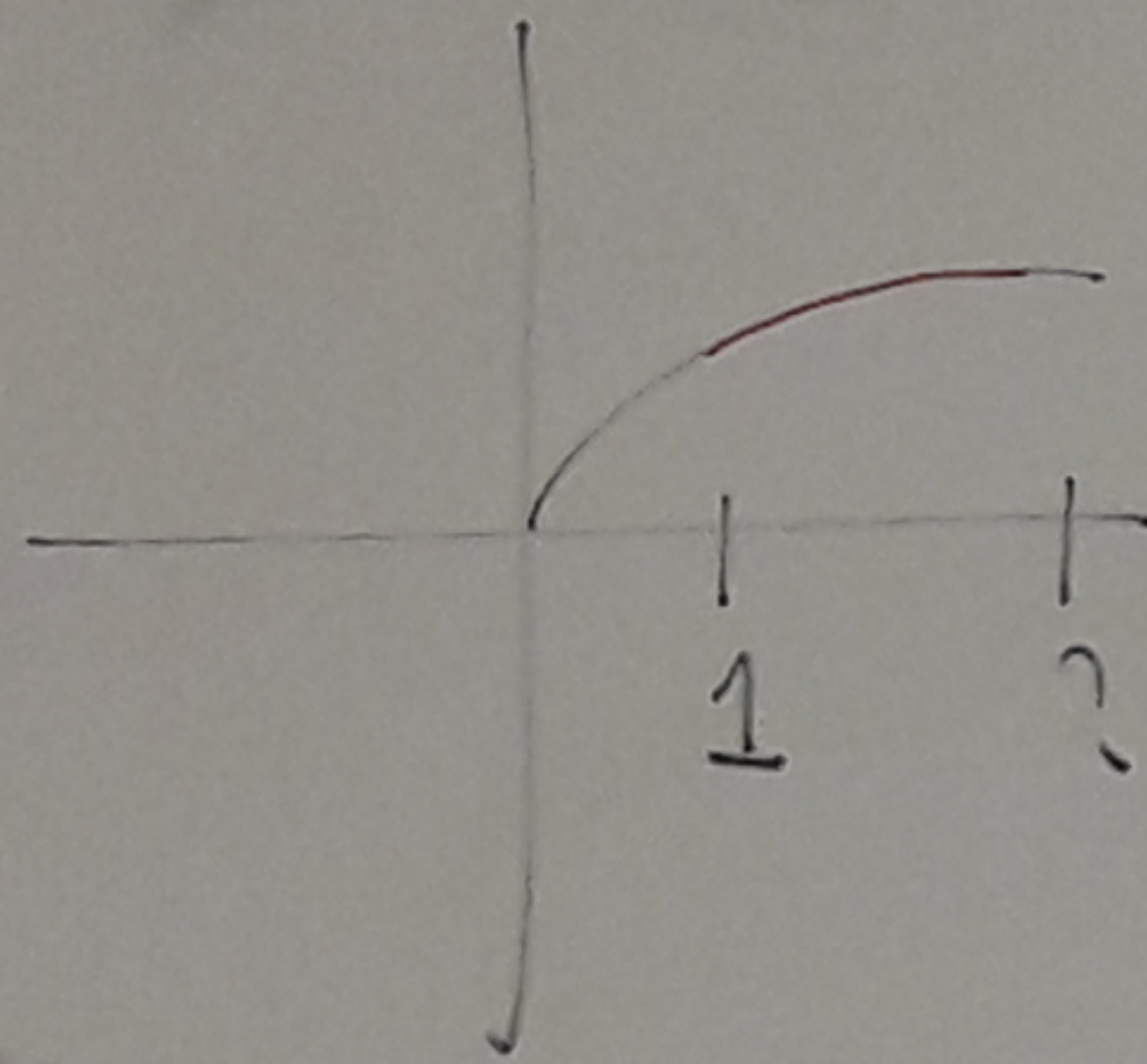


örnek 12.2

$$y = \sqrt{x}$$

$$1 \leq x \leq 2$$

Yay uzunluğu ?



$$y' = \frac{1}{2\sqrt{x}}$$

$$\int_1^2 \sqrt{1 + (y')^2} dx$$

$$= \int_1^2 \sqrt{1 + \frac{1}{4x}} dx$$

$$x = \frac{\tan^2 \sigma}{4}$$

2.1.3 Parametrik Eğriler.

$E \subset \mathbb{R}$ aralık olsun.

$\phi: E \rightarrow \mathbb{R}^n$ ϕ' fonksiyon.

(PARAMETRİZASYON)

$$S = \{ \phi(x) : x \in E \} \subset \mathbb{R}^n$$

parametrik eğri denir.

örnek 2.1.4

a) $S = \{ (\cos \theta, \sin \theta) : \theta \in [0, 2\pi] \}$

b) Not: Aynı eğri için birden fazla parametrisasyon olabilir.

$$S = \{ (\cos \theta^2, \sin \theta^2) : \theta \in [0, \sqrt{2\pi}] \}$$

$$= 12 \int_0^{\pi/2} \sin \theta \cos \theta d\theta = 6$$

Çark 1 Yarı çap. 1) $(0, 0)$ metre/saniye A' şeklinde ilerliyor.
A nokta 10 saniye de ne kadar
yol alır?

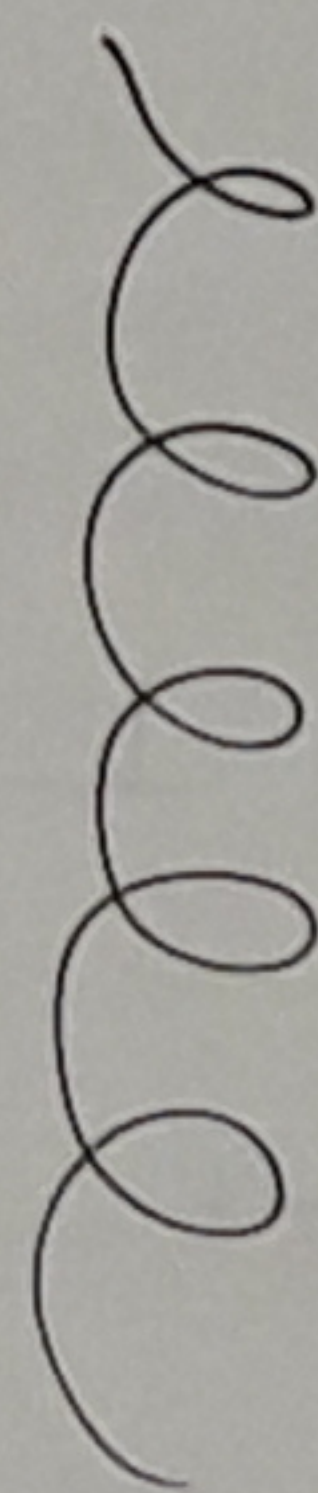
c) Astroid için parametvizasyon

$$S = \{ (\cos^3 \theta, \sin^3 \theta) : \theta \in [0, 2\pi] \}$$

d)

$$H = \{ (\sin x, \cos x, x) : x \in \mathbb{R} \}$$

Helix



parametrik eğriler için yay uzunluğu

$$S = \{ \phi(x) : x \in E \}$$

$\phi: E \rightarrow \mathbb{R}^n$ e' fonksiyon.

W(S) :

$$\int_E \|\phi'(t)\| dt$$

$$= \int_E \sqrt{\sum_{i=1}^n |\phi_i(t)|^2} dt$$

$$\phi = (\phi_1, \dots, \phi_n)$$

$$\phi' = (\phi'_1, \dots, \phi'_n)$$

Örnek 2.1.4

$$S = \left\{ (\cos^3 \theta, \sin^3 \theta) : \theta \in [0, 2\pi] \right\},$$

$$L(S) = \int_0^{2\pi} \sqrt{(3\cos^2 \theta \sin \theta)^2 + (3\sin^2 \theta \cos \theta)^2} d\theta$$

$$= \int_0^{2\pi} 3|\sin \theta \cos \theta| d\theta$$

$$= 12 \int_0^{\pi/2} \sin \theta \cos \theta d\theta = 6$$

$$H = \left\{ (\sin x, \cos x, x) : x \in [0, 2\pi] \right\}.$$

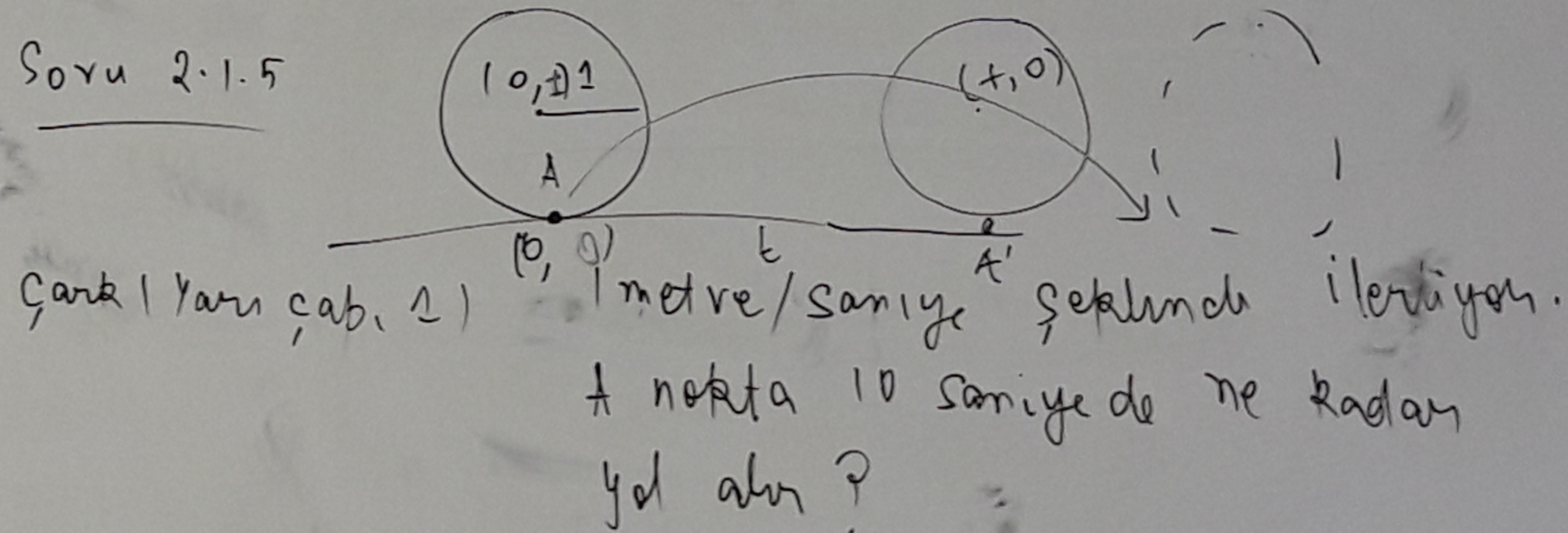
$$L(H) = \int_0^{2\pi} \sqrt{2} dx = 2\pi\sqrt{2}.$$

$$\varphi(x) = (\cos x, \sin x, x)$$

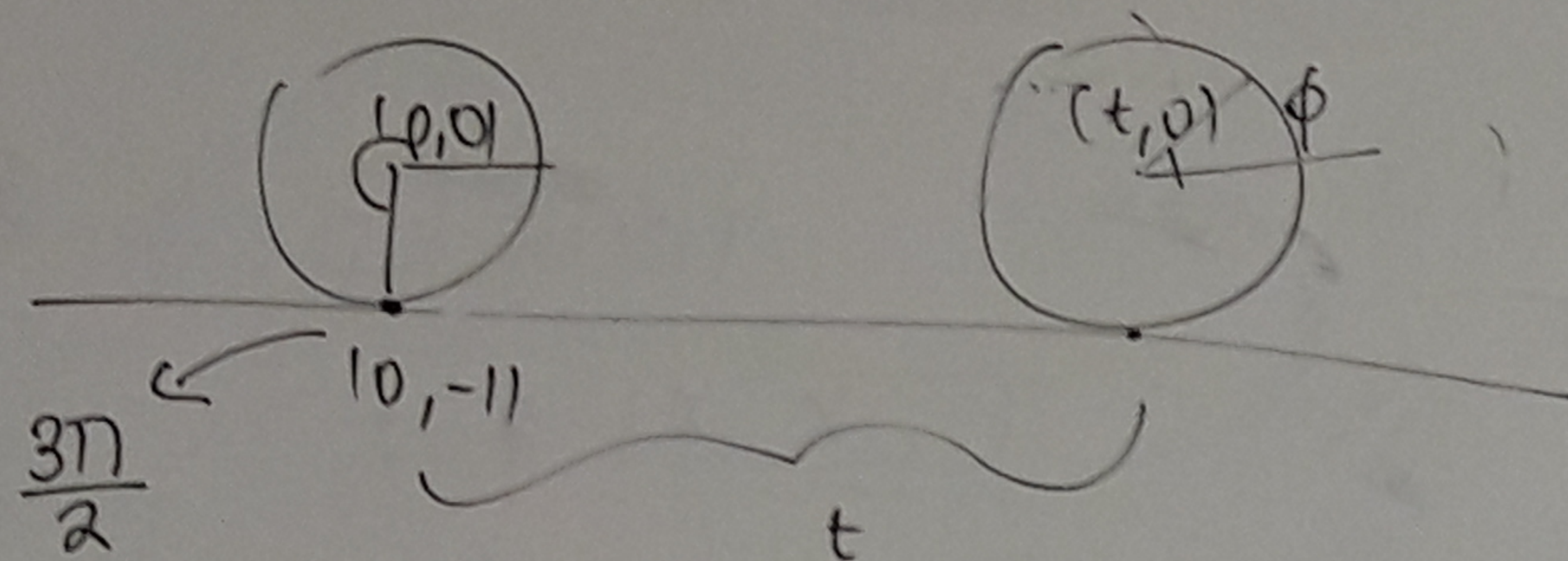
$$\varphi'(x) = (-\sin x, \cos x, 1)$$

$$\|\varphi'(x)\| = \sqrt{\sin^2 x + \cos^2 x + 1} = \sqrt{2}.$$

Soru 2.1.5



t saniye sonra



Çark: θ açısı ile döndü

θ zaman $\theta = t$

$(0, -1)$: $\theta = \frac{3\pi}{2}$

t saniye sonra θ : $\frac{3\pi}{2} + \theta = \frac{3\pi}{2} + t$

$(0, -1)$ 'nin yeni yeri:

$$\left(\cos\left(\frac{3\pi}{2} + t\right), \sin\left(\frac{3\pi}{2} + t\right) \right) + (t, 0)$$

$$= \left(t + \cos\left(\frac{3\pi}{2} + t\right), \sin\left(\frac{3\pi}{2} + t\right) \right)$$

$$= (t + \sin t, -\cos t)$$

CYCLOID

Yol uzunluğu

$$\int_0^t \sqrt{2} \sqrt{1 + \cos t} dt$$

$$d(t) = (t + \sin t, -\cos t)$$

$$d'(t) = (1 + \cos t, \sin t)$$

$$\begin{aligned} \|d'(t)\| &= \sqrt{1 + 2\cos t + \cos^2 t + \sin^2 t} \\ &= \sqrt{2} \sqrt{1 + \cos t} \end{aligned}$$

(7)

$$= \sqrt{2} \sqrt{1 + \cos t}$$

$$\cos t = 2 \cos^2\left(\frac{t}{2}\right) - 1$$

$$\int_0^{10} \sqrt{2} \cdot \sqrt{2} \left| \cos\left(\frac{t}{2}\right) \right| dt$$

$$= 2 \int_0^{10} \left| \cos\left(\frac{t}{2}\right) \right| dt$$

$$= \int_0^{10} |\cos(t)| dt$$

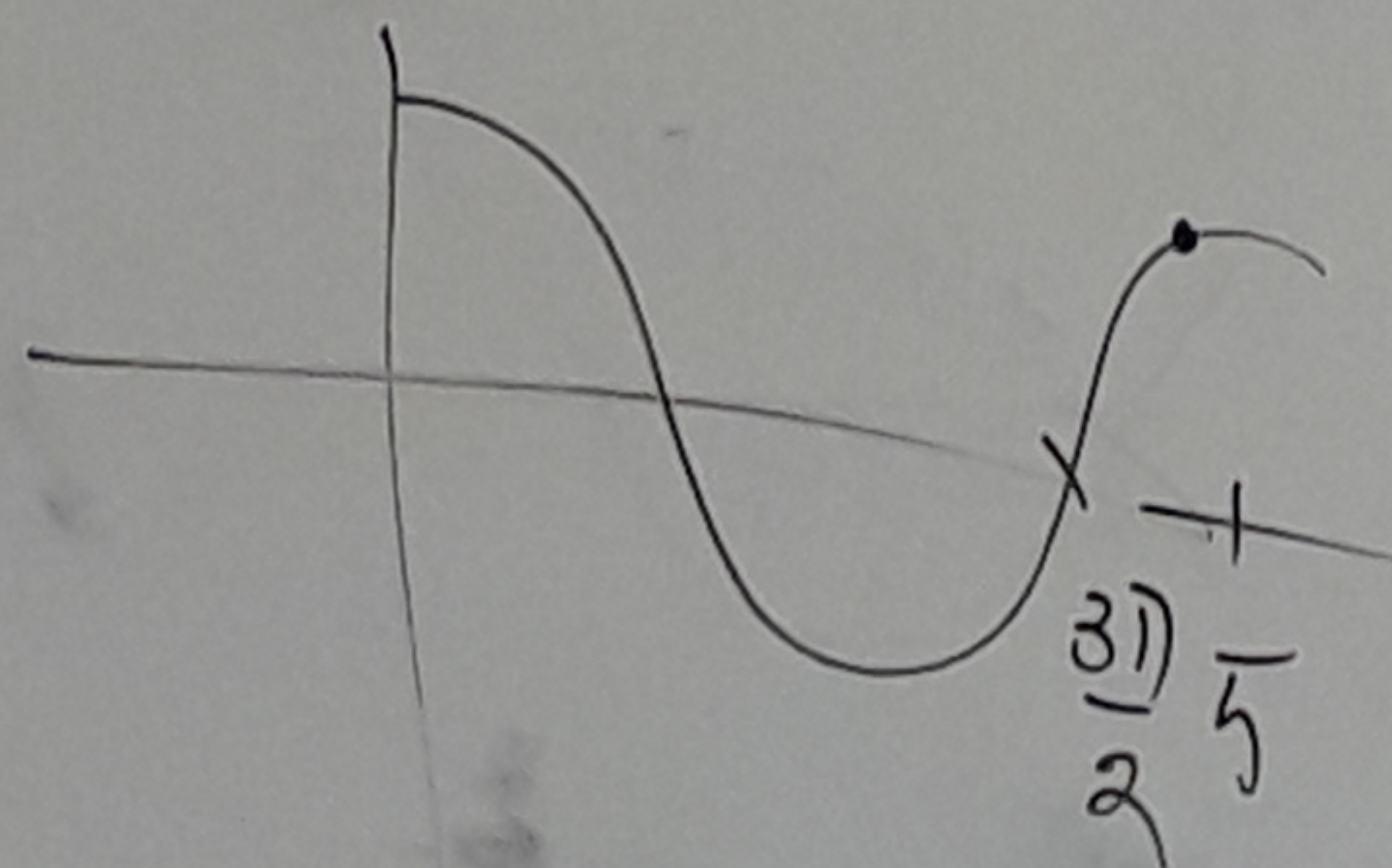
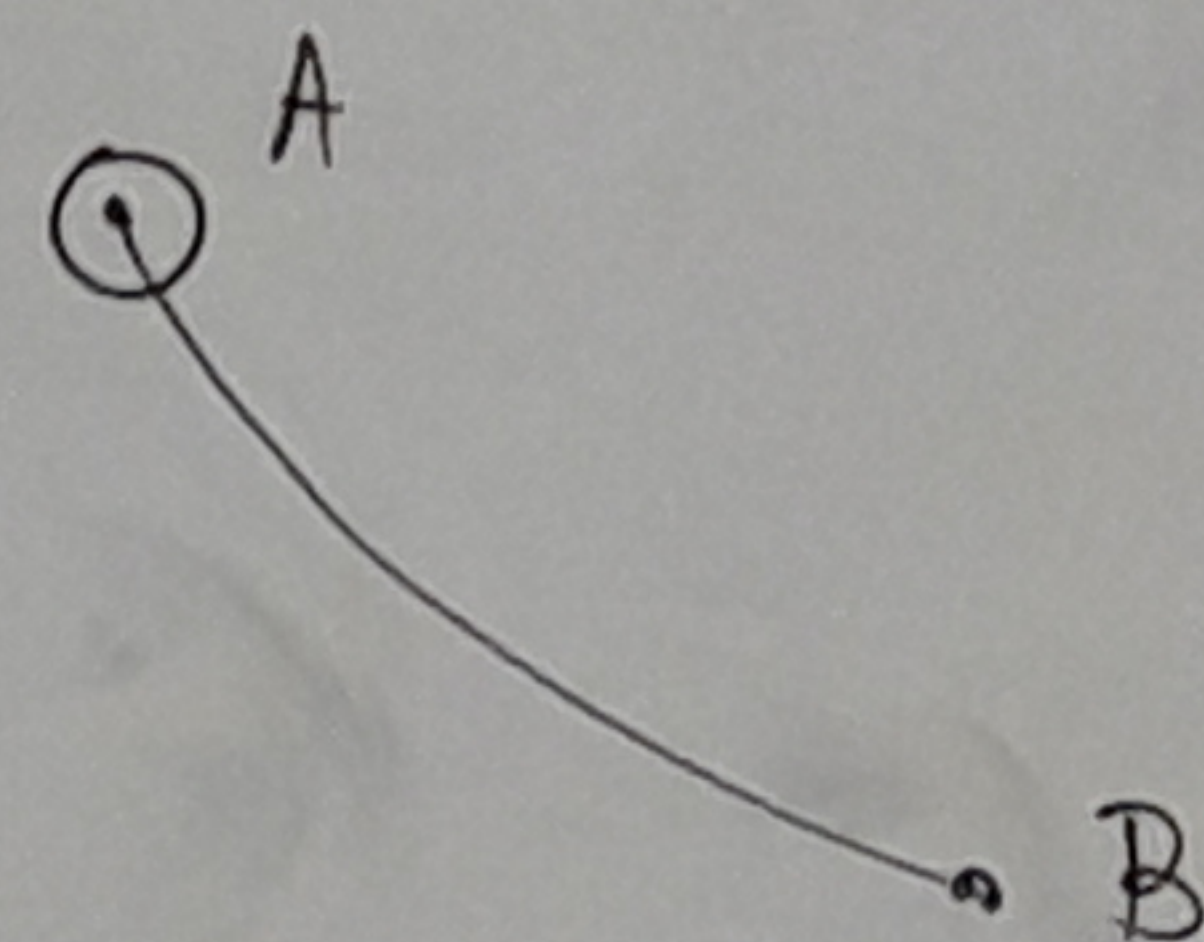
$$= 3 + \int_{\frac{3\pi}{2}}^{\pi} \left| \cos\left(\frac{t}{2}\right) \right| dt$$

$$= 3 + 1 - \sin(\pi)$$

$$= 4 - \sin(\pi)$$

(8)

BRACHISTOCHRONE



$$S = \{(x, y) \mid x+y \leq 1, x \geq 0, y \geq 0\}$$

$$S = \{(x, y, 1-x-y) \mid x \geq 0, y \geq 0, x+y \leq 1\}$$

Simpleks

0 zaman $S = \{(x, f(x)) : x \in E\}$ nın

Yüzey alanı

$$|S| := \int_E \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} dx dy$$

$|S| = \frac{|A|}{\cos \theta}$
 $\cos \theta = \frac{1}{\sqrt{1+f_x^2+f_y^2}}$

Teorem 8.1.6

$S \subset \mathbb{R}^n$ bir eğri olsun.

$\phi: E \rightarrow \mathbb{R}^n$ bir e' fonksiyon

$\psi: E \rightarrow \mathbb{R}^n$ olsun.

($E, \mathcal{I}$ aralıklar)

$S = \phi(E) = \psi(F)$ olsun. 0 zaman

$$\int_E \|\phi'(x)\| dx = \int_F \|\psi'(x)\| dx$$

2.1.7 Yüzeyler

Araf Yüzeyler.

$E \subset \mathbb{R}^2$ Açık küme

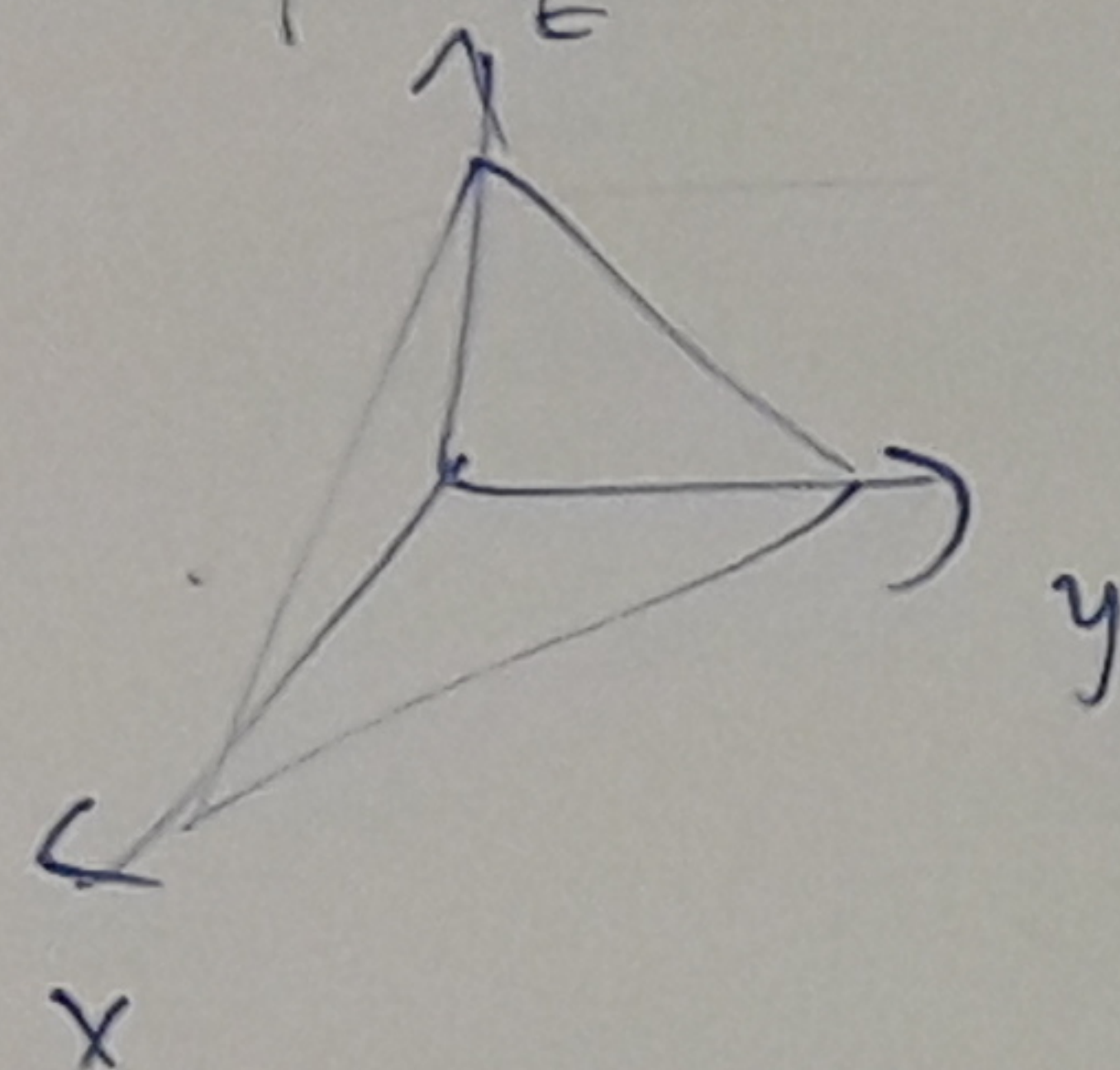
$f: E \rightarrow \mathbb{R}$.

$$\{(x, f(x)) : x \in E\} \subset \mathbb{R}^3$$

bir graf yüzey denir.

Örnekler. 2.1.8

a) Simpleks.



$$f(x, y) = 1 - x - y$$

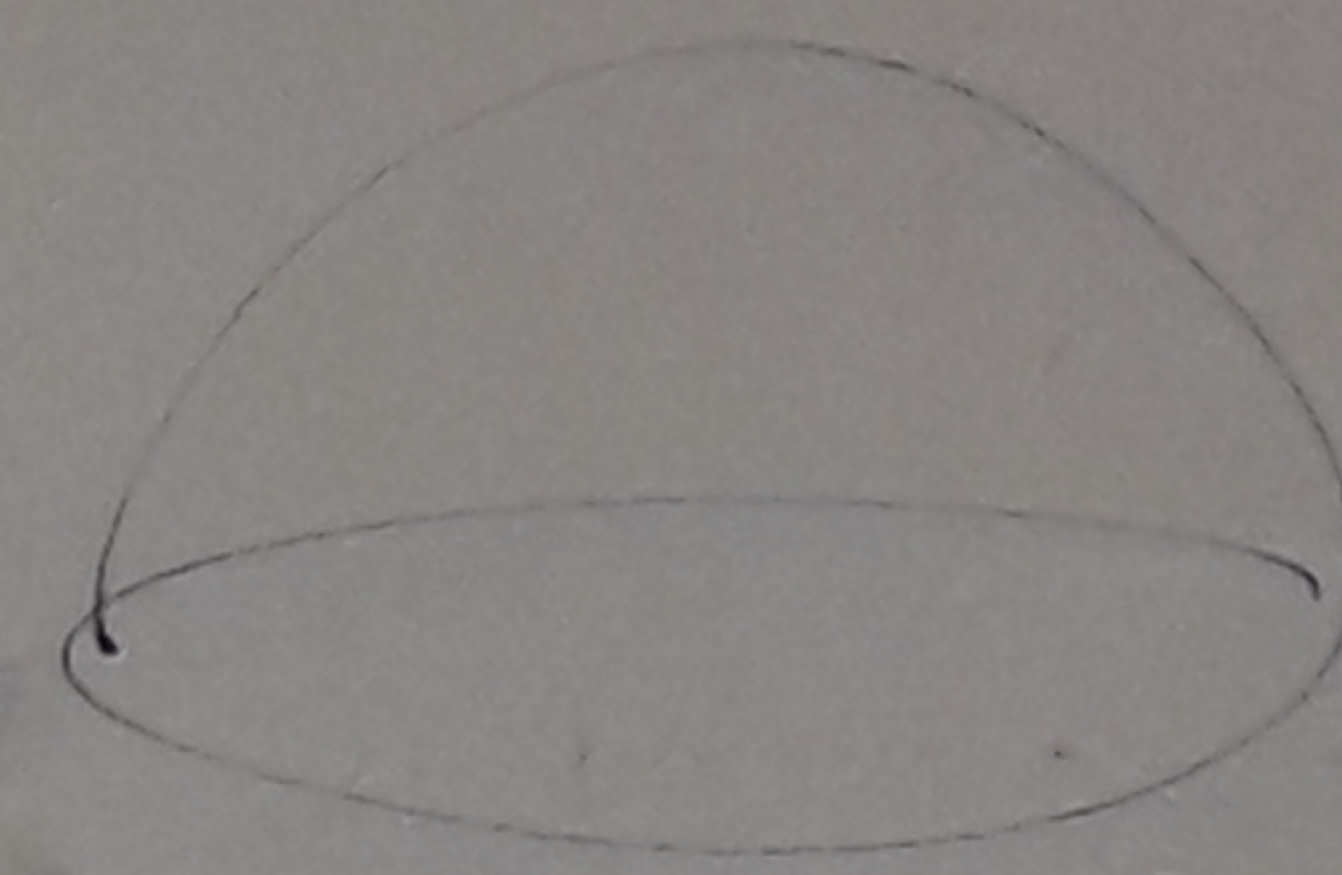
$$E = \{(x, y) \mid x + y \leq 1, x \geq 0, y \geq 0\}$$

$$S = \{(x, y, 1 - x - y) \mid x \geq 0, y \geq 0, x + y \leq 1\}$$

Simpleks

b) Yarı Top

$$\{(x, y, \sqrt{1 - x^2 - y^2}) \mid x^2 + y^2 \leq 1\}$$



$$(0, 0, 1)$$

2.1.9

Yüzey'in Yüzey Alan

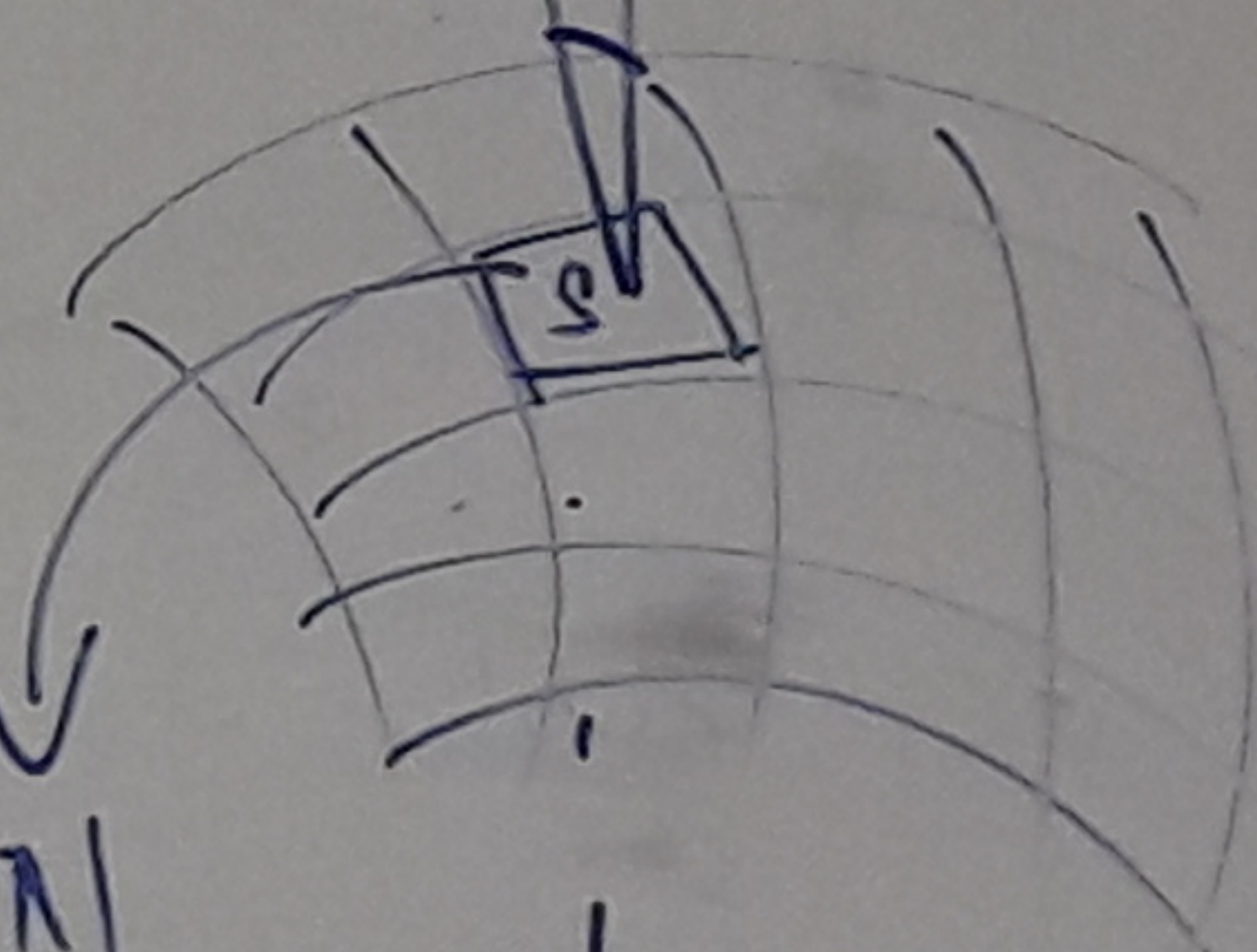
$E \subset \mathbb{R}^2$ açık küme

$f: E \rightarrow \mathbb{R}$ e' olsun.

O zaman $S = \{(x, f(x)) \mid x \in E\}$ nin Yüzey alan

$$|S| := \int_E \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} dx dy$$

$$\theta \uparrow \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, 1 \right)$$



$$|S| = |N|$$

$$\cos \theta$$

$$|R|$$

$$\cos \theta = \frac{1}{\sqrt{1 + f_x^2 + f_y^2}}$$

Örnek 2.1.9

S^2 'nin yüzey alanı

üst
Yarı $(x, y, \sqrt{1-x^2-y^2} : x^2+y^2 \leq 1)$

$$|S^2| = 2 \times \int \sqrt{1+f_x^2+f_y^2}$$

$$= 2 \int_0^{2\pi} \int_0^1 \frac{r dr d\theta}{\sqrt{1-r^2}} \quad \begin{matrix} x^2+y^2 \leq 1 \\ \theta=0 \quad r=0 \end{matrix}$$

$$f(x, y) = \sqrt{1-x^2-y^2}$$

$$f_x(x, y) = \frac{-x}{\sqrt{1-x^2-y^2}}$$

$$f_y(x, y) = \frac{-y}{\sqrt{1-x^2-y^2}}$$

$$\sqrt{1+f_x^2+f_y^2} = \sqrt{\frac{1-x^2-y^2+x^2+y^2}{1-x^2-y^2}} = \sqrt{\frac{1}{1-x^2-y^2}}$$

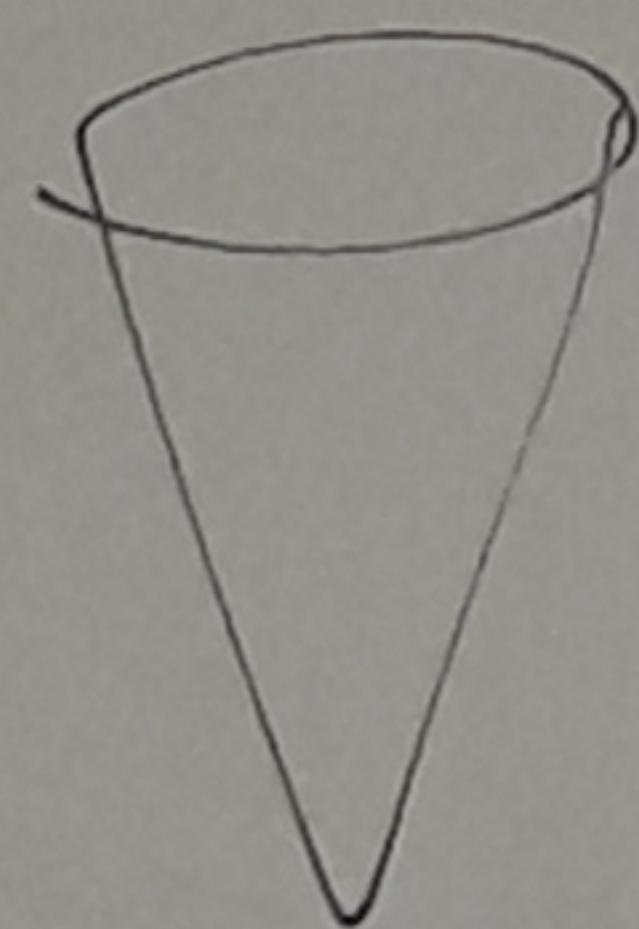
$$\rightarrow = 4\pi \left[-(1-r^2)^{1/2} \right]_0^1 = 4\pi \quad \square$$

$$\theta = 0 \quad r = 0 \quad \rightarrow \quad 4\pi \int_0^1 - (1-r^2) dr = 4\pi \left[-r + \frac{r^3}{3} \right]_0^1 = 4\pi \left(-1 + \frac{1}{3} \right) = -\frac{8\pi}{3}$$

örnek 2.1.1b

$$S = \left\{ (x, y, \sqrt{x^2 + y^2}) : x^2 + y^2 \leq 1 \right\}$$

Yüzey Alan?



$$f = \sqrt{x^2 + y^2}$$

$$f_x = \frac{x}{\sqrt{x^2 + y^2}}$$

$$f_y = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\sqrt{1 + f_x^2 + f_y^2} = \sqrt{1 + \frac{x^2}{x^2 + y^2} + \frac{y^2}{x^2 + y^2}} = \sqrt{2}$$

$$|S| = \int_{x^2 + y^2 \leq 1} \sqrt{2} = \pi \sqrt{2}$$

(12)