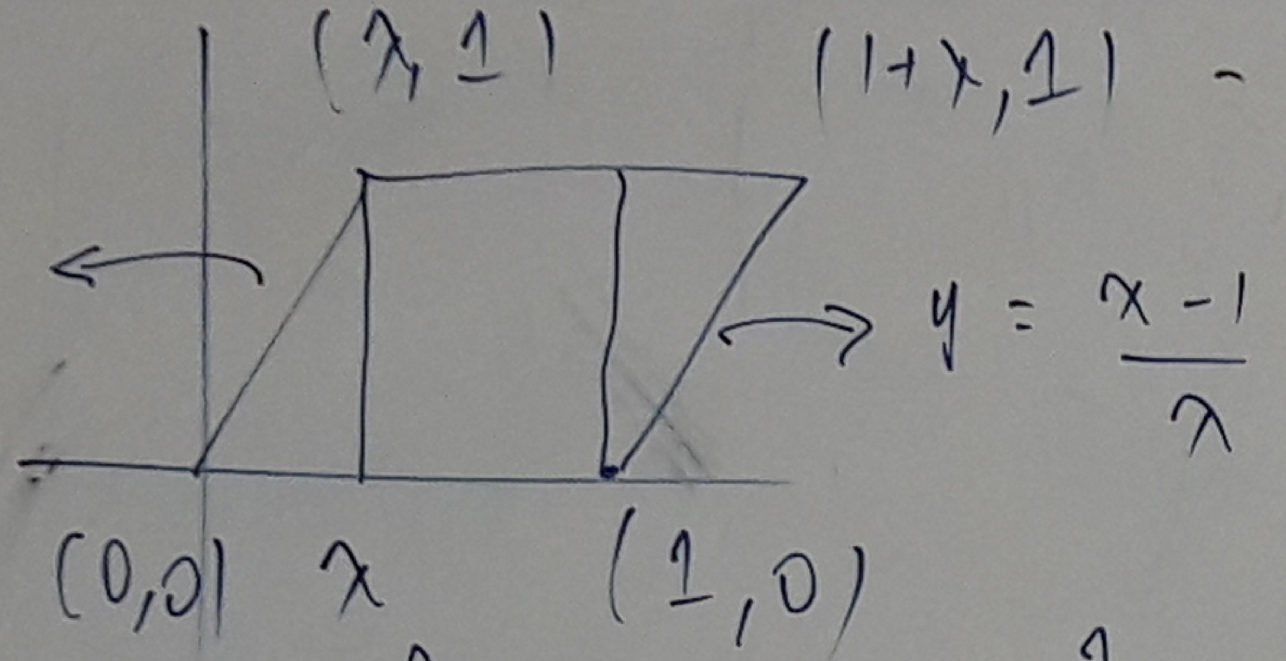
T(E) E İlişkisi.

6

$(\lambda+1,1)$



$\lambda \in [0,1]$ olsun

$$\int_S 1 = \int_{x=0}^{\lambda} \frac{x}{\lambda}\,dx + \int_{x=\lambda}^{1} 1\,dx + \int_{x=1}^{1+\lambda} \left(1 - \frac{x-1}{\lambda}\right) dx$$

$$= \left.\frac{x^2}{2\lambda}\right|_0^{\lambda} + (1-\lambda) + \left. x - \frac{x^2}{2\lambda} + \frac{x}{\lambda} \right|_1^{1+\lambda}$$

$$= \frac{\lambda}{2} + 1 - \cancel{\lambda} + \cancel{\lambda} - \frac{(1+\lambda)^2}{2\lambda} + \frac{1}{2\lambda} + 1$$

$$= 1 + \cancel{\frac{\lambda}{2}} + \cancel{\frac{1}{2\lambda}} - \cancel{\frac{1}{2\lambda}} + \cancel{\lambda} - \cancel{\frac{\lambda}{2}} + \cancel{1} = 1.$$

7

Sonuç $|T(R)| = |T(S)| = 1$.

Bu özel Matrisleri ve Köşegen Matrisler için Lemma İspatladık.

Çarpma kullanarak her matris için ispatlanmış olduk.

Lemma 1.4.2

$E \subset \mathbb{R}^n$ Jordan Bölge $T: \mathbb{R}^n \to \mathbb{R}^n$ lineer dönüşüm. O zaman

$$|T(E)| = |\det(T)|\,|E|.$$

$$= \frac{x}{2} + 1 - \cancel{x} + \cancel{x} - \frac{(1+x)^2}{2x} + \frac{1}{2x} + 1$$

$$= 1 + \frac{x}{2} + \frac{1}{2x} - \frac{1}{2x} + 1 - \frac{x}{2} + 1 = 1.$$

$E \subset \mathbb{R}^n$ Jordan Bölge $T: \mathbb{R}^n \to \mathbb{R}^n$ lineer dönüşüm. O zaman

$$|T(E)| = |\det(T)|\,|E|.$$

Lemma 1.4.3

$E \subset \mathbb{R}^n$ Jordan Bölge

$T: \mathbb{R}^n \longrightarrow \mathbb{R}^n$ lineer dönüşüm

$f: T(E) \longrightarrow \mathbb{R}$ integrellenebilir fonksiyon. O zaman,

$$\int_{T(E)} f = \int_E f \circ T \,|\det T|$$

İspat f: Integrellenebilir ise $f \geq 0$ (8)

$$\mathrm{Graf}_E(f) = \{(x,y) : x \in E,\ 0 \leq y \leq f(x)\}.$$

bir Jordan Bölgedir ve

$$|\mathrm{Graf}_E(f)| = \int_E f$$

$\mathrm{Graf}_E(f \circ T)$ ve $\mathrm{Graf}_{T(E)}(f)$ arasındaki

İlişkisi.

için göstermek yeter. S Paralel kenarın hacmi 1 dir.

$(x, f(T(x))) \longrightarrow (T(x), f(T(x)))$

Bu Lineer.

Matris : $S = \left[\begin{array}{c|c} T & 0 \\ \hline 0 & 1 \end{array}\right] \left[\begin{array}{c} x \\ \hline f(T(x)) \end{array}\right] = \left[\begin{array}{c} T(x) \\ \hline f(T(x)) \end{array}\right]$

$\det(S) = \det(T)$.

Not $S\left[\text{Graf}_E\, f\circ T\right] = \text{Graf}_{T(E)}\, f$ $\quad \det(T)$

Lemma 1.4.2 $\longrightarrow$ $\left|\text{Graf}_E\, f\circ T\right| \, |\det(S)| = \left|\text{Graf}_{T(E)}\, f\right|$

$\Rightarrow \left(\int_E f\circ T\right) |\det(T)| = \int_{T(E)} f$ $\quad$ II.

Değişken Değişiklik Formula

Teorem 1.4.4 $\quad E \subset \mathbb{R}^n$ Jordan

$\phi : E \longrightarrow \mathbb{R}^n$ $\mathcal{C}^1$ ve 1-1

$f : \phi(E) \longrightarrow \mathbb{R}$ Integrellenebilir

O zaman

$$\int_E (f\circ\phi)|\Delta_\phi| = \int_{\phi(E)} f$$

$\Delta_\phi = \det \phi'$

E $\phi(E)$

$\Delta_\phi := \det \phi'$

Örnek 1.4.5 Polar ko-ordinat.

$E \subset \mathbb{R}^2$

$x = r\cos\theta$

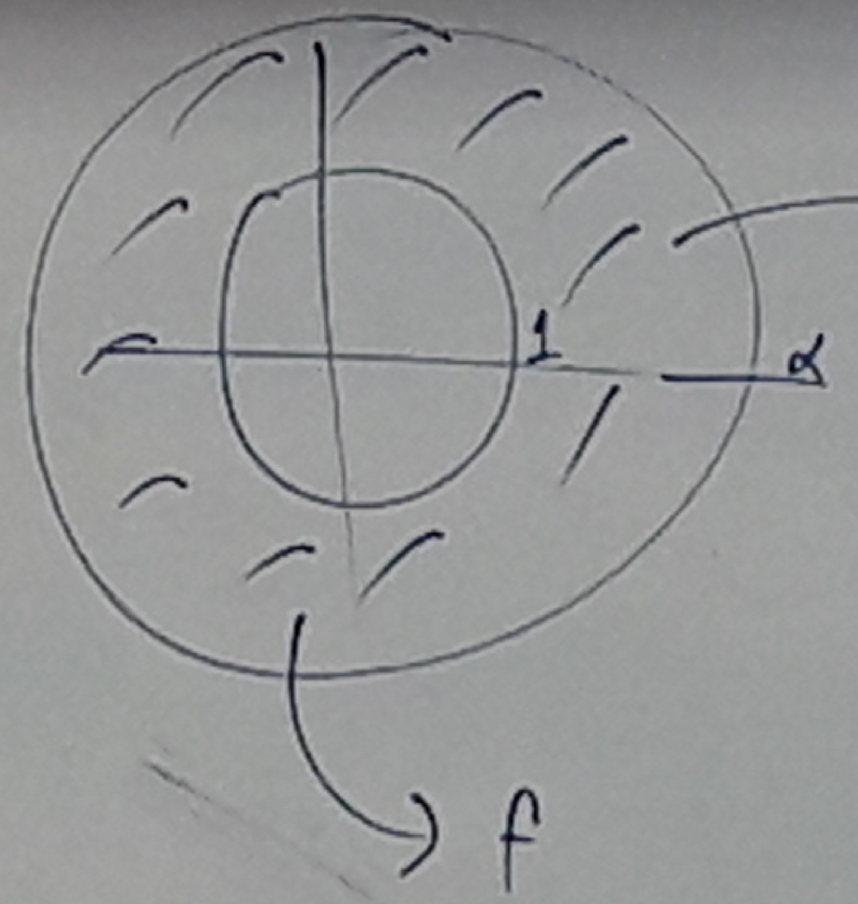
$y = r\sin\theta$

$$F := \{ (r,\theta) \mid (r\cos\theta, r\sin\theta) \in E \}.$$

$$\phi(r,\theta) = (\overset{u}{r\cos\theta}, \overset{v}{r\sin\theta})$$

$$\phi'(r,\theta) = \begin{bmatrix} \frac{\partial u}{\partial r} & \frac{\partial u}{\partial \theta} \\ \frac{\partial v}{\partial r} & \frac{\partial v}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix}$$

$$\Delta_\phi(r,\theta) = |\det \phi'(r,\theta)| = r.$$



(10)

$$E \quad \{ (x,y) \mid 1 \le x^2 + y^2 \le \alpha^2 \}$$

$$F \quad \left\{ (r,\theta) \;\middle|\; \begin{array}{l} 1 \le r \le \alpha \\ 0 \le \theta < 2\pi \end{array} \right\}$$

$$\int_E f(x,y)\,dx\,dy \qquad \int_F f(r,\theta) \boxed{?}\, dr\, d\theta$$

(11)

Polar-Koordinat formula.

$$\int_E f(x,y)\,dx\,dy = \int_F f(r,\theta)\cdot r\cdot dr\,d\theta$$

$$\int_{x^2+y^2\leq 1} x^2y^2\,dx\,dy = \int_{0\leq\theta\leq 2\pi}\int_{r=0}^{1} r^4\cos^2\theta\sin^2\theta\cdot r\,dr\,d\theta$$

$$= \frac{1}{5}\int_0^{2\pi}\frac{1-\cos 4\theta}{8}\,d\theta$$

$$= \frac{\pi}{20}.$$