MODEL-THEORY OF LIE-RINGS

DAVID PIERCE

The prototypical Lie-ring is $(\mathrm{End}(E), \circ - \circ')$, where E is an abelian group, $\mathrm{End}(E)$ is the abelian group of its endomorphisms, $\circ$ is composition, and $x \circ' y = y \circ x$. Then an arbitrary ring $(E, \cdot)$ is a Lie-ring if

$$x \cdot y + y \cdot x = 0,$$
$$(x \cdot y) \cdot z = x \cdot (y \cdot z) - y \cdot (x \cdot z).$$

The latter identity means that the map $u \mapsto (z \mapsto u \cdot z)$ is a homomorphism from $(E, \cdot)$ to $(\mathrm{End}(E), \circ - \circ')$. Commutative rings then have a parallel definition: $x \cdot y - y \cdot x = 0$, and $u \mapsto (z \mapsto u \cdot z)$ is a homomorphism from $(E, \cdot)$ to $(\mathrm{End}(E), \circ)$. Other rings can be defined in terms of different linear combinations $*$ of $\circ$ and $\circ'$, but they don't behave so nicely: we do not in general have that $u \mapsto (z \mapsto u * z)$ is a homomorphism from $(\mathrm{End}(E), *)$ to $(\mathrm{End}(\mathrm{End}(E)), *)$ unless $*$ is $\circ$ or $\circ - \circ'$ or trivial.

To what extent will the model-theory of Lie-rings parallel that of commutative rings (integral domains, fields)? The set of derivations of a commutative ring is naturally a Lie-ring. Hence, for example, the model-theory of differential fields gives rise to model-complete and ω-stable theories of (expansions of) Lie-rings.

Mathematics Department, Middle East Technical University, Ankara 06531, Turkey

E-mail address: dpierce@math.metu.edu.tr

URL: http://www.math.metu.edu.tr/~dpierce/