SELECTIONS

ILLUSTRATING THE HISTORY OF

GREEK MATHEMATICS

WITH AN ENGLISH TRANSLATION BY

IVOR THOMAS

FORMERLY SCHOLAR OF ST. JOHN'S AND SENIOR DEMY
OF MAGDALEN COLLEGE, OXFORD

IN TWO VOLUMES

II

FROM ARISTARCHUS TO PAPPUS

LONDON

WILLIAM HEINEMANN LTD

CAMBRIDGE, MASSACHUSETTS

HARVARD UNIVERSITY PRESS

MCMLVII

φέρειαν. ὃν δὲ λόγον ἔχει ἁ ΚΜΡ ποτὶ τὰν ΚΜΔ περιφέρειαν, τοῦτον ἔχει ἁ ΧΑ ποτὶ ΑΔ· ἐλάσσονα ἄρα λόγον ἔχει ἁ ΕΑ ποτὶ ΑΡ ἢ ἁ ΑΧ ποτὶ ΔΑ· ὅπερ ἐστὶν ἀδύνατον. οὐκ ἄρα μείζων ἁ ΖΑ τᾶς ΚΜΔ περιφερείας. ὁμοίως δὲ τοῖς πρότερον δειχθήσεται, ὅτι οὐδὲ ἐλάσσων ἐστίν· ἴσα ἄρα.

(*f*) Semi-Regular Solids

Papp. *Coll.* v. 19, ed. Hultsch i. 352. 7–354. 10

Πολλὰ γὰρ ἐπινοῆσαι δυνατὸν στερεὰ σχήματα παντοίας ἐπιφανείας ἔχοντα, μᾶλλον δ' ἄν τις ἀξιώσειε λόγου τὰ τετάχθαι δοκοῦντα [καὶ τούτων πολὺ πλέον τούς τε κώνους καὶ κυλίνδρους καὶ τὰ καλούμενα πολύεδρα].[1] ταῦτα δ' ἐστὶν οὐ μόνον τὰ παρὰ τῷ θειοτάτῳ Πλάτωνι πέντε σχήματα, τουτέστιν τετράεδρόν τε καὶ ἑξάεδρον, ὀκτάεδρόν τε καὶ δωδεκάεδρον, πέμπτον δ' εἰκοσάεδρον, ἀλλὰ καὶ τὰ ὑπὸ Ἀρχιμήδους εὑρεθέντα τρισκαίδεκα τὸν ἀριθμὸν ὑπὸ ἰσοπλεύρων μὲν καὶ ἰσογωνίων οὐχ ὁμοίων δὲ πολυγώνων περιεχόμενα.

[1] καὶ . . . πολύεδρα om. Hultsch.

---

[a] This part of the proof involves a *verging* assumed in Prop. 8, just as the earlier part assumed the *verging* of Prop. 7. The verging of Prop. 8 has already been described (vol. i. p. 350 n. *b*) in connexion with Pappus's comments on it.

[b] Archimedes goes on to show that the theorem is true even if the tangent touches the spiral in its second or some higher turn, not at the extremity of the turn ; and in Props. 18 and 19 he has shown that the theorem is true if the tangent should touch at an extremity of a turn.

Now arc KMP : arc KMΔ = XA : AΔ ; [Prop. 14

∴ EA : AP < AX : ΔA ;

which is impossible. Therefore ZA is not greater than the arc KMΔ. In the same way as above it may be shown to be not less [a]; therefore it is equal.[b]

(*f*) Semi-Regular Solids

Pappus, *Collection* v. 19, ed. Hultsch i. 352. 7–354. 10

Although many solid figures having all kinds of surfaces can be conceived, those which appear to be regularly formed are most deserving of attention. Those include not only the five figures found in the godlike Plato, that is, the tetrahedron and the cube, the octahedron and the dodecahedron, and fifthly the icosahedron,[c] but also the solids, thirteen in number, which were discovered by Archimedes [d] and are contained by equilateral and equiangular, but not similar, polygons.

As Pappus (ed. Hultsch 302. 14-18) notes, the theorem can be established without recourse to propositions involving *solid loci* (for the meaning of which see vol. i. pp. 348-349), and proofs involving only " plane " methods have been developed by Tannery, *Mémoires scientifiques*, i., 1912, pp. 300-316 and Heath, *H.G.M.* ii. 556-561. It must remain a puzzle why Archimedes chose his particular method of proof, especially as Heath's proof is suggested by the figures of Props. 6 and 9 ; Heath (*loc. cit.*, p. 557) says " it is scarcely possible to assign any reason except his definite predilection for the form of proof by *reductio ad absurdum* based ultimately on his famous ' Lemma ' or Axiom."

[c] For the five regular solids, see vol. i. pp. 216-225.

[d] Heron (*Definitions* 104, ed. Heiberg 66. 1-9) asserts that two were known to Plato. One is that described as P_2 below, but the other, said to be bounded by eight squares and six triangles, is wrongly given.

Τὸ μὲν γὰρ πρῶτον ὀκτάεδρόν ἐστιν περιεχόμενον ὑπὸ τριγώνων δ καὶ ἑξαγώνων δ.

Τρία δὲ μετὰ τοῦτο τεσσαρεσκαιδεκάεδρα, ὧν τὸ μὲν πρῶτον περιέχεται τριγώνοις η̄ καὶ τετραγώνοις ς̄, τὸ δὲ δεύτερον τετραγώνοις ς̄ καὶ ἑξαγώνοις η̄, τὸ δὲ τρίτον τριγώνοις η̄ καὶ ὀκταγώνοις ς̄.

Μετὰ δὲ ταῦτα ἑκκαιεικοσάεδρά ἐστιν δύο, ὧν τὸ μὲν πρῶτον περιέχεται τριγώνοις η̄ καὶ τετραγώνοις ιη̄, τὸ δὲ δεύτερον τετραγώνοις ιβ̄, ἑξαγώνοις η̄ καὶ ὀκταγώνοις ς̄.

Μετὰ δὲ ταῦτα δυοκαιτριακοντάεδρά ἐστιν τρία, ὧν τὸ μὲν πρῶτον περιέχεται τριγώνοις κ̄ καὶ πενταγώνοις ιβ̄, τὸ δὲ δεύτερον πενταγώνοις ιβ̄ καὶ ἑξαγώνοις κ̄, τὸ δὲ τρίτον τριγώνοις κ̄ καὶ δεκαγώνοις ιβ̄.

Μετὰ δὲ ταῦτα ἕν ἐστιν ὀκτωκαιτριακοντάεδρον περιεχόμενον ὑπὸ τριγώνων λβ̄ καὶ τετραγώνων ς̄.

Μετὰ δὲ τοῦτο δυοκαιεξηκοντάεδρά ἐστι δύο, ὧν τὸ μὲν πρῶτον περιέχεται τριγώνοις κ̄ καὶ τετραγώνοις λ̄ καὶ πενταγώνοις ιβ̄, τὸ δὲ δεύτερον τετραγώνοις λ̄ καὶ ἑξαγώνοις κ̄ καὶ δεκαγώνοις ιβ̄.

Μετὰ δὲ ταῦτα τελευταῖόν ἐστιν δυοκαιενενηκοντάεδρον, ὃ περιέχεται τριγώνοις π̄ καὶ πενταγώνοις ιβ̄.

---

[a] For the purposes of n. *b*, the thirteen polyhedra will be designated as $P_1, P_2 \ldots P_{13}$.

[b] Kepler, in his *Harmonice mundi* (*Opera*, 1864, v. 123-126), appears to have been the first to examine these figures systematically, though a method of obtaining some is given in a scholium to the Vatican ms. of Pappus. If a solid angle of a regular solid be cut by a plane so that the same length is cut off from each of the edges meeting at the solid angle,

The first is a figure of eight bases, being contained by four triangles and four hexagons [P_1].[a]

After this come three figures of fourteen bases, the first contained by eight triangles and six squares [P_2], the second by six squares and eight hexagons [P_3], and the third by eight triangles and six octagons [P_4].

After these come two figures of twenty-six bases, the first contained by eight triangles and eighteen squares [P_5], the second by twelve squares, eight hexagons and six octagons [P_6].

After these come three figures of thirty-two bases, the first contained by twenty triangles and twelve pentagons [P_7], the second by twelve pentagons and twenty hexagons [P_8], and the third by twenty triangles and twelve decagons [P_9].

After these comes one figure of thirty-eight bases, being contained by thirty-two triangles and six squares [P_{10}].

After this come two figures of sixty-two bases, the first contained by twenty triangles, thirty squares and twelve pentagons [P_{11}], the second by thirty squares, twenty hexagons and twelve decagons [P_{12}].

After these there comes lastly a figure of ninety-two bases, which is contained by eighty triangles and twelve pentagons [P_{13}].[b]

the section is a regular polygon which is a triangle, square or pentagon according as the solid angle is composed of three, four or five plane angles. If certain equal lengths be cut off in this way from all the solid angles, regular polygons will also be left in the faces of the solid. This happens (i) obviously when the cutting planes bisect the edges of the solid, and (ii) when the cutting planes cut off a smaller length from each edge in such a way that a regular polygon is left in each face with double the number of sides. This method gives (1) from the tetrahedron, P_1; (2) from the

(g) System of expressing Large Numbers

Archim. *Aren.* 3, Archim. ed. Heiberg ii. 236. 17–240. 1

Ἃ μὲν οὖν ὑποτίθεμαι, ταῦτα· χρήσιμον δὲ εἶμεν ὑπολαμβάνω τὰν κατονόμαξιν τῶν ἀριθμῶν ῥηθῆμεν, ὅπως καὶ τῶν ἄλλων οἱ τῷ βιβλίῳ μὴ περιτετευχότες τῷ ποτὶ Ζεύξιππον γεγραμμένῳ μὴ πλανῶνται διὰ τὸ μηδὲν εἶμεν ὑπὲρ αὐτᾶς ἐν τῷδε τῷ βιβλίῳ προειρημένον. συμβαίνει δὴ τὰ ὀνόματα τῶν ἀριθμῶν ἐς τὸ μὲν τῶν μυρίων ὑπάρχειν ἁμῖν παραδεδομένα, καὶ ὑπὲρ τὸ τῶν μυρίων [μὲν][1] ἀποχρεόντως γιγνώσκομες μυριάδων ἀριθμὸν λέγοντες ἔστε ποτὶ τὰς μυρίας μυριάδας. ἔστων οὖν ἁμῖν οἱ μὲν νῦν εἰρημένοι ἀριθμοὶ ἐς τὰς μυρίας μυριάδας πρῶτοι καλουμένοι, τῶν δὲ πρώτων ἀριθμῶν αἱ μύριαι μυριάδες μονὰς καλείσθω δευτέρων ἀριθμῶν, καὶ ἀριθμείσθων τῶν δευτέρων μονάδες καὶ ἐκ τᾶν μονάδων δεκάδες καὶ ἑκατοντάδες καὶ χιλιάδες καὶ μυριάδες ἐς τὰς μυρίας μυριάδας. πάλιν δὲ καὶ αἱ μύριαι μυριάδες τῶν δευτέρων ἀριθμῶν μονὰς καλείσθω τρίτων ἀριθμῶν, καὶ ἀριθμείσθων τῶν τρίτων ἀριθμῶν μονάδες καὶ ἀπὸ τᾶν μονάδων δεκάδες καὶ ἑκατοντάδες καὶ χιλιάδες καὶ μυριάδες ἐς τὰς μυρίας μυριάδας. τὸν αὐτὸν δὲ τρόπον καὶ τῶν τρίτων ἀριθμῶν μύριαι μυριάδες μονὰς καλείσθω τετάρτων ἀριθμῶν,

[1] μὲν om. Heiberg.

---

cube, P_3 and P_4; (3) from the octahedron, P_2 and P_3; (4) from the icosahedron, P_7 and P_8; (5) from the dodecahedron, P_7 and P_9. It was probably the method used by Plato.

Four more of the semi-regular solids are obtained by first cutting all the edges symmetrically and equally by planes parallel to the edges, and then cutting off angles. This

(*g*) System of expressing Large Numbers

Archimedes, *Sand-Reckoner* 3, Archim. ed. Heiberg ii. 236. 17–240. 1

Such are then the assumptions I make; but I think it would be useful to explain the naming of the numbers, in order that, as in other matters, those who have not come across the book sent to Zeuxippus may not find themselves in difficulty through the fact that there had been no preliminary discussion of it in this book. Now we already have names for the numbers up to a myriad [10^4], and beyond a myriad we can count in myriads up to a myriad myriads [10^8]. Therefore, let the aforesaid numbers up to a myriad myriads be called *numbers of the first order* [numbers from 1 to 10^8], and let a myriad myriads of numbers of the first order be called a unit of *numbers of the second order* [numbers from 10^8 to 10^{16}], and let units of the numbers of the second order be enumerable, and out of the units let there be formed tens and hundreds and thousands and myriads up to a myriad myriads. Again, let a myriad myriads of numbers of the second order be called a unit of *numbers of the third order* [numbers from 10^{16} to 10^{24}], and let units of numbers of the third order be enumerable, and from the units let there be formed tens and hundreds and thousands and myriads up to a myriad myriads. In the same manner, let a myriad myriads of numbers of the third order be

gives (1) from the cube, P_5 and P_6; (2) from the icosahedron, P_{11}; (3) from the dodecahedron, P_{12}.

The two remaining solids are more difficult to obtain; P_{10} is the *snub cube* in which each solid angle is formed by the angles of four equilateral triangles and one square; P_{13} is the *snub dodecahedron* in which each solid angle is formed by the angles of four equilateral triangles and one regular pentagon.