Abscissas and Ordinates

David Pierce

June 2, 2014, 9:57 a.m.
Mathematics Department
Mimar Sinan Fine Arts University
Istanbul
dpierce@msgsu.edu.tr
http://mat.msgsu.edu.tr/~dpierce/

In the manner of Apollonius of Perga, but hardly any modern book, we investigate conic sections *as such.* We thus discover why Apollonius calls a conic section a parabola, an hyperbola, or an ellipse; and we discover the meanings of the terms abscissa and ordinate. In an education that is liberating and not simply indoctrinating, the student of mathematics will learn these things.

Contents

1 The liberation of mathematics

In the first of the eight books of the *Conics* [3], Apollonius of Perga derives properties of the conic sections that can be used to write their equations in rectangular or oblique coordinates.[1] This article reviews these properties, because (1) they have intrinsic mathematical interest, (2) they are the reason why Apollonius gave to the three conic sections the names that they now have, and (3) the vocabulary of Apollonius is a source for many of our technical terms.

In a modern textbook of analytic geometry, the two coordinates of a point in the so-called Cartesian plane may be called the "abscissa" and "ordinate." Probably the book will not explain why. But the reader deserves an explanation. The student should not have to learn meaningless words, for the same reason that s/he should not be expected to memorize the quadratic formula without a derivation. True education is not indoctrination, but liberation. Mathematics is

[1] The first four books of the *Conics* survive in Greek; the next three, in Arabic translation only. The last book is lost. Lucio Russo [22, p. 8] uses this and other examples to show that we cannot expect the best ancient work to have survived.

liberating when it teaches us our own power to decide what is true. This power comes with a responsibility to justify our decisions to anybody who asks; but this is a responsibility that must be shared by all of us who do mathematics.

Mathematical terms *can* be assigned arbitrarily. This is permissible, but it is not desirable. The terms "abscissa" and "ordinate" arise quite naturally in Apollonius's development of the conic sections. This development should be better known, especially by anybody who teaches analytic geometry. This is why I write.

2 Lexica and registers

Apollonius did not create his terms: they are just ordinary words, used to refer to mathematical objects. When we do not *translate* Apollonius, but simply transliterate his words, or use their Latin translations, then we put some distance between ourselves and the mathematics. When I first learned that a conic section had a *latus rectum,* I had a sense that there was a whole theory of conic sections that was not being revealed, although its existence was hinted at by this peculiar Latin term. If we called the *latus rectum* by its English name of "upright side," then the student could ask, "What is upright about it?" In turn, textbook writers might feel obliged to answer this question. In any case, I am going to answer it here. Briefly, it is called upright because, for good reason, it is to be conceived as having one endpoint on the vertex of the conic section, but as sticking out from the plane of the section.

English does borrow foreign words freely: this is a characteristic of the language. A large lexicon is not a bad thing. A choice from among two or more synonyms can help establish the register of a piece of speech. In the 1980s, as I recall, there was a book called *Color Me Beautiful* that was on the American bestseller lists week after week. The *New York Times* blandly said the book provided "beauty tips for women"; the *Washington Post* described it as offering "the color-wheel approach to female pulchritude." By using an obscure synonym for beauty, the *Post* mocked the book.

If distinctions between near-synonyms are maintained, then subtleties of expression are possible. "Circle" and "cycle" are Latin and Greek words for the same thing, but the Greek word is used more abstractly in English, and it would be bizarre to refer to a finite group of prime order as being circular rather than cyclic.

To propose or maintain distinctions between near-synonyms is a *raison d'être* of works like Fowler's *Dictionary of Modern English Usage* [10]. Fowler laments, for example, the use of the Italian word *replica* to refer to any copy of an art-work, when the word properly refers to a copy *made by the same artist.* In his article on synonyms, Fowler sees in language the kind of liberation, coupled with responsibility, that I ascribed to mathematics:

> Synonym books in which differences are analysed, engrossing as they may have been to the active party, the analyst, offer to the passive party, the reader, nothing but boredom. Every reader must, for the most part, be his own analyst; & no-one who does not expend, whether expressly & systematically or as a half-conscious accompaniment of his reading & writing, a good deal of care upon points of synonymy is likely to write well.

The boredom of the reader of a book of synonyms may be comparable to that of the reader of a mathematics textbook that begins with a bunch of strange words like "abscissa" and "ordinate."

Mathematics can be done in any language. Greek does mathematics without a specialized vocabulary. It is worthwhile to consider what this is like.

I shall take Apollonius's terminology from Heiberg's edition [2] (actually a printout of a `pdf` image downloaded from the Wilbour Hall website, `wilbourhall.org`). Meanings are checked with the big Liddell–Scott–Jones lexicon [15] (available from the Perseus Digital

Library, `perseus.tufts.edu`, though I splurged on the print version myself).

I am going to write out Apollonius's terms in Greek letters. I shall use the customary minuscule forms developed in the Middle Ages. Apollonius himself would have used only the letters that we now call capital; but modern mathematics uses minuscule Greek letters freely, and the reader ought to be able to make sense of them.

3 The gendered Greek article

Apollonius's word for **cone** is ὁ κῶνος, meaning originally "pine-cone." Evidently our word comes ultimately from Apollonius's (and this is confirmed by such resources as [12]). I write out the ὁ to indicate the gender of κῶνος: ὁ is the masculine definite article. The feminine article is ἡ. In each case, the diacritical mark over the vowel indicates the prefixed sound that is spelled in English with the letter H. Other diacritical marks can be ignored; I reproduce them because they are in the modern texts.

In the terminology of Apollonius, all of the nouns that we shall look at will be feminine or masculine. Greek does however have a neuter gender as well,[2] and the neuter article is τό. I want to note by the way the economy of expression that is made possible by gendered articles. In mathematics, **point** is τὸ σημεῖον, neuter; **line** is ἡ γραμμή, feminine. The feminine ἡ στιγμή can also be used for a mathematical point; it is *not used,* argues Reviel Netz [20, p. 113], so that an expression like ἡ Α can unambiguously designate a particular *line* in a diagram, while τὸ Α would designate a point. In Proposition I.43 of the *Elements,* Euclid can refer to a parallelogram ΑΕΚΘ simply as τὸ ΕΘ [7, p. 100]: the neuter article is used, because παραλληλόγραμμον is neuter. The reader cannot think that τὸ ΕΘ is a line; the line would be ἡ ΕΘ. The English reader *can* make this mistake. In Heath's translation [9, 8], Euclid says,

[2]English retains the threefold gender distinction in "he/she/it."

> Let $ABCD$ be a parallelogram, and AC its diameter; and about AC let EH, FG be parallelograms, and BK, KD the so-called complements.

It is confusing to see both lines and parallelograms given two-letter designations. Perhaps the confusion is easily overcome; but the Greek reader would not have had it in the first place. This is one of the few cases where gender in a language is actually useful.

4 The cone of Apollonius

For Apollonius, a **cone** (ὁ κῶνος "pine-cone," as above) is a solid figure determined by (1) a **base** (ἡ βάσις), which is a circle, and (2) a **vertex** (ἡ κορυφή "summit"), which is a point that is not in the plane of the base. The surface of the cone contains all of the straight lines drawn from the vertex to the circumference of the base. A **conic surface** (ἡ κωνικὴ ἐπιφάνεια[3]) consists of such straight lines, not bounded by the base or the vertex, but extended indefinitely in both directions.

The straight line drawn from the vertex of a cone to the center of the base is the **axis** (ὁ ἄξων "axle") of the cone. If the axis is perpendicular to the base, then the cone is **right** (ὀρθός); otherwise it is **scalene** (σκαληνός "uneven"). Apollonius considers both kinds of cones indifferently.

A plane containing the axis intersects the cone in a triangle. Suppose a cone with vertex A has axial triangle ABC. Then the base BC of this triangle is a diameter of the base of the cone. Let an arbitrary chord[4] DE of the base of the cone cut the base BC of the axial triangle at right angles at a point F, as in Figure 1. In the

[3] The word ἐπιφάνεια means originally "appearance" and is the source of the English "epiphany."

[4] Although it is the source of the English "cord" and "chord" [12], Apollonius does not use the word ἡ χορδή, although he proves in Proposition I.10 that

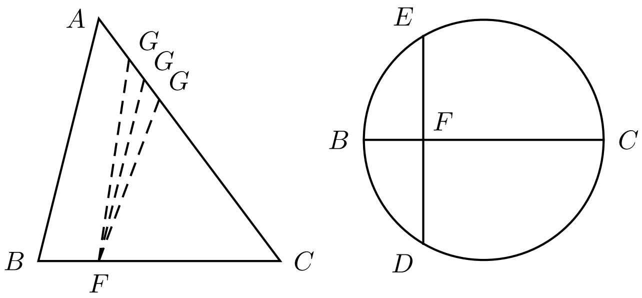

Figure 1 Axial triangle and base of a cone

axial triangle, let a straight line FG be drawn from the base to the side AC. This straight line FG may, but need not, be parallel to the side BA. It is not at right angles to DE, unless the plane of the axial triangle is at right angles to the plane of the base of the cone. In any case, the two straight lines FG and DE, meeting at F, are not in a straight line with one another, and so they determine a plane. This plane cuts the surface of the cone in such a curve DGE as is shown in Figure 2. Apollonius refers to such a curve first (in Proposition I.7) as a section (ἡ τομή) in the *surface* of the cone, and later (I.10) as a section of a cone. All of the chords of this section that are parallel to DE are bisected by the straight line GF. Therefore Apollonius calls this straight line a **diameter** (ἡ διάμετρος [γραμμή]) of the section.[5]

The parallel chords bisected by the diameter are said to be drawn to the diameter **in an orderly way.** The Greek adverb here is τε-

the straight line joining any two points of a conic section *is* a chord, in the sense that it falls within the section. The Greek χορδή means gut, hence *anything* made with gut, be it a lyre-string or a sausage [15].

[5]The associated verb is διαμετρέ-ω "measure through"; this is the verb used in Homer's *Iliad* [13, III.315]) for what Hector and Odysseus do in measuring out lists for the single combat of Paris and Menelaus. (The reference is in [15].)

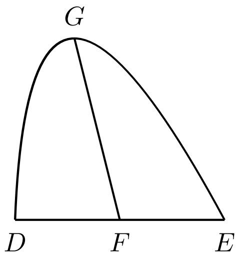

Figure 2 A conic section

ταγμένως, from the verb τάσσω, which has meanings like "to draw up in order of battle" [15]. A Greek noun derived from this verb is τάξις, which is found in English technical terms like "taxonomy" and "syntax" [16]. The Latin adverb corresponding to the Greek τεταγμένως is *ordinate* from the verb *ordino.* From the Greek expression for "the straight line drawn in an orderly way," Apollonius will elide the middle part, leaving "the in-an-orderly-way."[6] This term will refer to *half* of a chord bisected by a diameter. Similar elision in the Latin leaves us with the word **ordinate** for this half-chord [18]. In the *Geometry,* Descartes refers to ordinates as *[lignes] qui s'appliquent par ordre [au] diametre* [5, p. 328].

I do not know whether the classical *orders* of architecture—the Doric, Ionic, and Corinthian orders—are so called because of the mathematical use of the word "ordinate." But we may compare the ordinates of a conic section as in Figure 3 with the row of columns of a Greek temple, as in Figure 4.

Back in Figure 2, the point G at which the diameter GF cuts

[6] Heath [1, p. clxi] translates τεταγμένως as "ordinate-wise"; Taliaferro [3, p. 3], as "ordinatewise." But this usage strikes me as anachronistic. The term "ordinatewise" seems to mean "in the manner of an ordinate"; but ordinates are just what we are trying to define when we translate τεταγμένως.

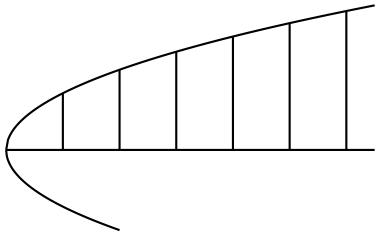

Figure 3 Ordinates of a conic section

the conic section DGE is called a **vertex** (κορυφής as before). The segment of the diameter between the vertex and an ordinate has come to be called in English an **abscissa;** but this just the Latin translation of Apollonius's Greek for being cut off (ἀπολαμβανομένη "taken"[7]).

Apollonius will show that every point of a conic section is the vertex for some unique diameter. If the ordinates corresponding to a particular diameter are at right angles to it, then the diameter will be an **axis** of the section. Meanwhile, in describing the relation between the ordinates and the abscissas of conic section, there are three cases to consider.

5 The parabola

Suppose the diameter of a conic section is parallel to a side of the corresponding axial triangle. For example, suppose in Figure 5 that FG is parallel to BA. The square on the ordinate DF is equal to the rectangle whose sides are BF and FC (by Euclid's Proposition III.35). More briefly, $DF^2 = BF \cdot FC$. But BF is independent of

[7] I note the usage of the Greek participle in [2, I.11, p. 38]. Its general usage for what we translate as *abscissa* is confirmed in [15], although the general sense of the verb is not of cutting, but of taking.

Figure 4 Columns in the Ionic order, at Priene, Söke, Aydın, Turkey

the choice of the point D on the conic section. That is, for any such choice (aside from the vertex of the section), a plane containing the chosen point and parallel to the base of the cone cuts the cone in another circle, and the axial triangle cuts this circle along a diameter, and the plane of the section cuts this diameter at right angles into two pieces, one of which is equal to BF. The square on DF thus varies as FC, which varies as FG. That is, the square on an ordinate varies as the abscissa (Apollonius I.20). Hence there is a straight line

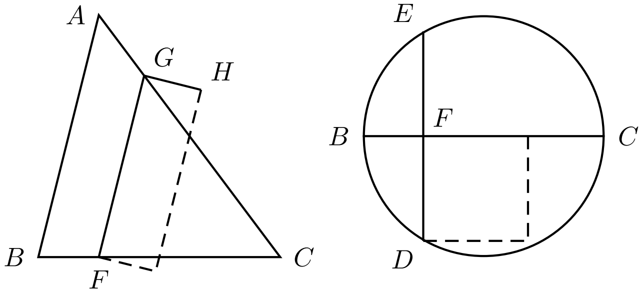

Figure 5

GH such that

$$DF^2 = FG \cdot GH,$$

and GH is independent of the choice of D.

This straight line GH can be conceived as being drawn at right angles to the plane of the conic section DGE. Therefore Apollonius calls GH the **upright side** (ὀρθία [πλευρά]), and Descartes accordingly calls it *le costé droit* [5, p. 329]. Apollonius calls the conic section itself ἡ παραβολή; we transliterate this as **parabola.** The Greek word is also the origin of the English "parable," but can have various related meanings, like "juxtaposition, comparison, conjunction, application." The word is self-descriptive: it can be understood as a juxtaposition of the preposition παρά "along, beside" and the noun ἡ βολή "throw." Alternatively, παραβολή is a noun derived from the verb παραβάλλω, which is παρά plus βάλλω "throw." In the parabola of Apollonius, the rectangle bounded by the abscissa and the upright side is the result of *applying* the square on the ordinate to the upright side. Such an application is made for example in Proposition I.44 of Euclid's *Elements,* where a parallelogram equal to a given triangle is *applied* to a given straight line: that is, the parallelogram is constructed on the given straight line as base.[8]

[8]This proposition is a lemma for Proposition 45, that if a figure with any

6 The *latus rectum*

The Latin term for the upright side is *latus rectum.* This term is also used in English. In the *Oxford English Dictionary* [18], the earliest quotation illustrating the use of the term is from a mathematical dictionary published in 1702. Evidently the quotation refers to Apollonius and gives his meaning:

> App. Conic Sections 11 In a Parabola the Rectangle of the Diameter, and Latus Rectum, is equal to the rectangle of the Segments of the double Ordinate.

I assume the "segments of the double ordinate" are the two halves of a chord, so that each of them is what we are calling an ordinate, and the rectangle contained by them is equal to the square on one of them.

The possibility of defining the conic sections in terms of a *directrix* and *focus* is shown by Pappus [21, VII.312–8, pp. 1004–15] and was presumably known to Apollonius. Pappus does not use such technical terms though; there is just a straight line and a point, as in the following, a slight modification[9] of Thomas's translation [24, pp. 492–503]:

> If AB be a straight line given in position, and the point Γ be given in the same plane, and ΔΓ be drawn, and ΔE be drawn perpendicular

number of straight sides be given, then a rectangle—or indeed a parallelogram in any given angle—can be constructed that is equal to this figure. This is the climax of Book I of the *Elements,* and it recalls Herodotus's tracing of the origins of geometry to the measuring of land lost in the annual flooding of the Nile in Egypt [11, II.109]. Propositions 47 and 48, the Pythagorean Theorem and its converse, are merely the *dénouement* of Book I of Euclid.

[9] I have put "the ratio of ΓΔ to ΔE" where Thomas has "the ratio ΓΔ : ΔE" because Pappus uses no special notation for a ratio as such, but refers merely to λόγος. . . τῆς ΓΔ πρὸς ΔE. The recognition of ratios as individual mathematical objects (namely numbers) distinguishes modern from ancient mathematics, although the beginnings of this recognition can be seen in Pappus; but that is a subject for another article.

> [to AB], and if the ratio of ΓΔ to ΔE be given, then the point Δ will lie on a conic section.[10]

A modern textbook may define the parabola in terms of a directrix and focus, explicitly so called. An example is Nelson, Folley, and Borgman, *Analytic Geometry* [19], a book that I happen to have on hand because my mother used it in college, and because I perused it at the age of 12 when I wanted to understand the curves that could be encoded in equations. Dissatisfaction with such textbooks leads me back to the Ancients. According to Nelson *& al.*,

> The chord of the parabola which contains the focus and is perpendicular to the axis is called the *latus rectum.* Its length is of value in estimating the amount of "spread" of the parabola.

The first sentence here defines the *latus rectum* as a certain line segment that is indeed equal to Apollonius's upright side. The second sentence correctly describes the significance of the *latus rectum.* However, the juxtaposition of the two sentences may mislead somebody who knows just a little Latin. The Latin adjective *latus, -a, -um* does mean "broad, wide; spacious, extensive" [17]: it is the root of the English noun "latitude." An extensive *latus rectum* does mean a broad parabola. However, the Latin adjective *latus* is unrelated to

[10] As Heath [1, pp. xxxvi–xl] explains, Pappus proves this theorem because Euclid did not supply a proof in his treatise on *surface loci.* (This treatise itself is lost to us.) Euclid must have omitted the proof because it was already well known; and Euclid predates Apollonius. Morris Kline [14, p. 96] summarizes all of this by saying that the focus-directrix property "was known to Euclid and is stated and proved by Pappus." Later (on his page 128), Kline gives a precise reference to Pappus: it is Proposition 238, in Hultsch's numbering, of Book VII. Actually this proposition is a recapitulation, which is incomplete in the extant manuscripts; one must read a few pages earlier in Pappus for more details, as in the selection in Thomas's anthology. In any case, Kline says, "As noted in the preceding chapter, Euclid probably knew" the proposition. According to Boyer however, "It appears that Apollonius knew of the focal properties for central conics, but it is possible that the focus-directrix property for the parabola was not known before Pappus" [4, §XI.12, p. 211].

the noun *latus, -eris* “side; flank,” which is found in English in the adjective “lateral”; and the noun *latus* is what is used in the phrase *latus rectum.*[11]

Denoting abscissa by x, and ordinate by y, and *latus rectum* by ℓ, we have for the parabola the modern equation

$$y^2 = \ell x. \tag{$*$}$$

The letters here can be considered as numbers in the modern sense, or just as line segments, or congruence-classes of segments.

[11] In *latus rectum,* the adjective *rectus, -a, -um* “straight, upright” is given the neuter form, because the noun *latus* is neuter. The plural of *latus rectum* is *latera recta.* The neuter plural of the adjective *latus* would be *lata.* The dictionary writes the adjective as *lātus,* with a long “a”; but the “a” in the noun is unmarked and therefore short. As far as I can tell, the adjective is to be distinguished from another Latin adjective with the same spelling (and the same long “a”), but with the meaning of “carried, borne”, used for the past participle of the verb *fero, ferre, tulī, lātum.* This past participle appears in English in words like “translate,” while *fer-* appears in “transfer.” The *American Heritage Dictionary* [16] traces *lātus* “broad” to an Indo-European root *stel-* and gives “latitude” and “dilate” as English derivatives; *lātus* “carried” comes from an Indo-European root *tel-* and is found in English words like “translate” and “relate,” but also “dilatory.” Thus “dilatory” is not to be considered as a derivative of “dilate.” A French etymological dictionary [6] implicitly confirms this under the adjacent entries *dilater* and *dilatoire.* The older Skeat [23] does give “dilatory” as a derivative of “dilate.” However, under “latitude,” Skeat traces *lātus* “broad” to the Old Latin *stlātus,* while under “tolerate” he traces *lātum* “borne” to *tlātum.* In his introduction, Skeat says he has collated his dictionary “with the *New English Dictionary* [as the *Oxford English Dictionary* was originally called] from A to H (excepting a small portion of G).” In fact the *OED* distinguishes *two* English verbs “dilate,” one for each of the Latin adjectives *lātus.* But the dictionary notes, “The sense ‘prolong’ comes so near ‘enlarge’, ‘expand’, or ‘set forth at length’. . . that the two verbs were probably not thought of as distinct words.”

7 The hyperbola

The second possibility for a conic section is that the diameter meets the other side of the axial triangle when this side is extended beyond the vertex of the cone. In Figure 6, the diameter FG, crossing one

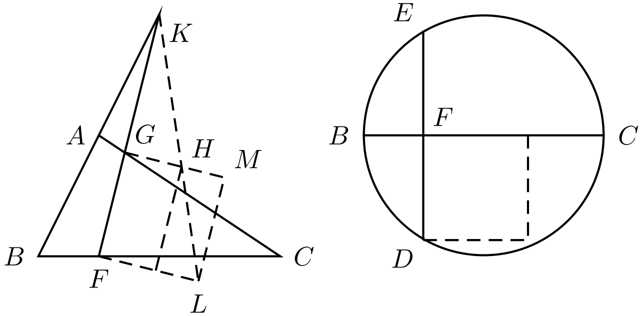

Figure 6

side of the axial triangle ABC at G, crosses the other side, extended, at K. Again $DF^2 = BF \cdot FC$; but the latter product now varies as $KF \cdot FG$. The upright side GH can now be defined so that

$$BF \cdot FC : KF \cdot FG :: GH : GK.$$

We draw KH and extend to L so that FL is parallel to GH, and we extend GH to M so that LM is parallel to FG. Then

$$\begin{aligned} FL \cdot FG : KF \cdot FG &:: FL : KF \\ &:: GH : GK \\ &:: BF \cdot FC : KF \cdot FG, \end{aligned}$$

and so $FL \cdot FG = BF \cdot FC$. Thus

$$DF^2 = FG \cdot FL.$$

Apollonius calls the conic section here an **hyperbola** (ἡ ὑπερβολή), that is, an *excess,* an *overshooting,* a *throw* (βολή) *beyond* (ὑπέρ), because the square on the ordinate is equal to a rectangle whose one side is the abscissa, and whose other side is applied to the upright side: but this rectangle *exceeds* (ὑπερβάλλω), by another rectangle, the rectangle contained by the abscissa and the upright side. The excess rectangle is similar to the rectangle contained by the upright side GH and GK. Apollonius calls GK the **transverse side** (ἡ πλαγία πλευρά) of the hyperbola. Denoting it by a, and the other segments as before, we have the modern equation

$$y^2 = \ell x + \frac{\ell}{a}x^2. \qquad (\dagger)$$

8 The ellipse

The last possibility is that the diameter meets the other side of the axial triangle when this side is extended below the base. All of the computations will be as for the hyperbola, except that now, if it is considered as a *directed* segment, the transverse side is negative, and so the modern equation is

$$y^2 = \ell x - \frac{\ell}{a}x^2. \qquad (\ddagger)$$

In this case Apollonius calls the conic section an **ellipse** (ἡ ἔλλειψις), that is, a *falling short,* because again the square on the ordinate is equal to a rectangle whose one side is the abscissa, and whose other side is applied to the upright side: but this rectangle now *falls short* (ἐλλείπω) of the rectangle contained by the abscissa and the upright side by another rectangle. Again this last rectangle is similar to the rectangle contained by the upright and transverse sides.

9 Descartes

We have seen that the terms "abscissa" and "ordinate" are ultimately translations of Greek words that describe certain line segments determined by points on conic sections. For Apollonius, an ordinate and its corresponding abscissa are not required to be at right angles to one another.

Descartes generalizes the use of the terms slightly. In one example [5, p. 339], he considers a curve derived from a given conic section in such a way that, if a point of the conic section is given by an equation of the form

$$y^2 = \dots x \dots,$$

then a point on the new curve is given by

$$y^2 = \dots x' \dots,$$

where xx' is constant. But Descartes just describes the new curve in words:

> toutes les lignes droites appliquées par ordre a son diametre estant esgales a celles d'une section conique, les segmens de ce diametre, qui sont entre le sommet & ces lignes, ont mesme proportion a une certaine ligne donnée, que cete ligne donnée a aux segmens du diametre de la section conique, auquels les pareilles lignes sont appliquées par ordre.[12]

The new curve has ordinates, namely *les lignes droites appliqués par ordre a son diametre.* These ordinates have corresponding abscissas, *les segmens de ce diametre, qui sont entre le sommet & ces lignes.* There is still no notion that an arbitrary point might have

[12] "All of the straight lines drawn in an orderly way to its diameter being equal to those of a conic section, the segments of this diameter that are between the vertex and these lines have the same ratio to a given line that this given line has to the segments of the diameter of the conic section to which the parallel lines are drawn in an orderly way."

two coordinates, called abscissa and ordinate respectively. A point determines an ordinate and abscissa only insofar as the point belongs to a given curve with a designated diameter.

References

[1] Apollonius of Perga, *Apollonius of Perga: Treatise on conic sections*, University Press, Cambridge, UK, 1896, Edited by T. L. Heath in modern notation, with introductions including an essay on the earlier history of the subject.

[2] ———, *Apollonii Pergaei qvae Graece exstant cvm commentariis antiqvis*, vol. I, Teubner, 1974, Edidit et Latine interpretatvs est I. L. Heiberg.

[3] ———, *Conics. Books I–III*, revised ed., Green Lion Press, Santa Fe, NM, 1998, Translated and with a note and an appendix by R. Catesby Taliaferro, with a preface by Dana Densmore and William H. Donahue, an introduction by Harvey Flaumenhaft, and diagrams by Donahue, edited by Densmore. MR MR1660991 (2000d:01005)

[4] Carl B. Boyer, *A history of mathematics*, John Wiley & Sons, New York, 1968.

[5] René Descartes, *The geometry of René Descartes*, Dover Publications, Inc., New York, 1954, Translated from the French and Latin by David Eugene Smith and Marcia L. Latham, with a facsimile of the first edition of 1637.

[6] Jean Dubois, Henri Mitterand, and Albert Dauzat, *Dictionnaire d'étymologie*, Larousse, Paris, 2001, First edition 1964.

[7] Euclid, *Euclidis Elementa*, Euclidis Opera Omnia, vol. I, Teubner, 1883, Edidit et Latine interpretatvs est I. L. Heiberg.

[8] ———, *The thirteen books of Euclid's Elements translated from the text of Heiberg. Vol. I: Introduction and Books I, II. Vol. II: Books III–IX. Vol. III: Books X–XIII and Appendix*, Dover Publications Inc., New York, 1956, Translated with introduction and commentary by Thomas L. Heath, 2nd ed. MR 17,814b

[9] ———, *Euclid's Elements*, Green Lion Press, Santa Fe, NM, 2002, All thirteen books complete in one volume. The Thomas L. Heath translation, edited by Dana Densmore. MR MR1932864 (2003j:01044)

[10] H. W. Fowler, *A dictionary of modern English usage*, Wordsworth Editions, Ware, Hertfordshire, UK, 1994, reprint of the original 1926 edition.

[11] Herodotus, *The Persian wars, books I–II*, Loeb Classical Library, vol. 117, Harvard University Press, Cambridge, Massachusetts and London, England, 2004, Translation by A. D. Godley; first published 1920; revised, 1926.

[12] T. F. Hoad (ed.), *The concise Oxford dictionary of English etymology*, Oxford University Press, Oxford and New York, 1986, Reissued in new covers, 1996.

[13] Homer, *The Iliad*, Loeb Classical Library, Harvard University Press and William Heinemann Ltd, Cambridge, Massachusetts, and London, 1965, with an English translation by A. T. Murray.

[14] Morris Kline, *Mathematical thought from ancient to modern times*, Oxford University Press, New York, 1972. MR MR0472307 (57 #12010)

[15] Henry George Liddell and Robert Scott, *A Greek-English lexicon*, Clarendon Press, Oxford, 1996, revised and augmented throughout by Sir Henry Stuart Jones, with the assistance of

Roderick McKenzie and with the cooperation of many scholars. With a revised supplement.

[16] William Morris (ed.), *The Grolier international dictionary*, Grolier Inc., Danbury, Connecticut, 1981, two volumes; appears to be the *American Heritage Dictionary* in a different cover.

[17] James Morwood (ed.), *The pocket Oxford Latin dictionary*, Oxford University Press, 1995, First edition published 1913 by Routledge & Kegan Paul.

[18] Murray et al. (eds.), *The compact edition of the Oxford English Dictionary*, Oxford University Press, 1973.

[19] Alfred L. Nelson, Karl W. Folley, and William M. Borgman, *Analytic geometry*, The Ronald Press Company, New York, 1949.

[20] Reviel Netz, *The shaping of deduction in Greek mathematics*, Ideas in Context, vol. 51, Cambridge University Press, Cambridge, 1999, A study in cognitive history. MR MR1683176 (2000f:01003)

[21] Pappus, *Pappus Alexandrini collectionis quae supersunt*, vol. II, Weidmann, Berlin, 1877, E libris manu scriptis edidit, Latina interpretatione et commentariis instruxit Fridericus Hultsch.

[22] Lucio Russo, *The forgotten revolution*, Springer-Verlag, Berlin, 2004, How science was born in 300 BC and why it had to be reborn, translated from the 1996 Italian original by Silvio Levy. MR MR2038833 (2004k:01006)

[23] Walter W. Skeat, *A concise etymological dictionary of the English language*, Perigee Books, New York, 1980, First edition 1882; original date of this edition not given.

[24] Ivor Thomas (ed.), *Selections illustrating the history of Greek mathematics. Vol. I. From Thales to Euclid*, Loeb Classical Library, vol. 335, Harvard University Press, Cambridge, Mass., 1951, With an English translation by the editor. MR 13,419a