Learning mathematics—
Some hints from the psychologists

In your degree course you will learn, and be examined on:

1. Facts (= 'knowing that')
2. Skills (= 'knowing how').

Examples:

An example of a fact to know is a trigonometric formula:

$$\sin(a+b) = \sin a \cos b + \cos a \sin b.$$

An example of a skill is solving simultaneous equations.

Learning facts

Hint 1.

As a human being, you have only one reliable way of learning facts:
Connect them to other facts and things.

The connections can be of any kind you find helpful:

1. picture images
2. sound and rhythm
3. logical structure (look for the words ‘and’, ‘not’, ‘if . . . then’, ‘all/every’, ‘there exists/for some’)
4. other mathematical ideas

We strongly recommend the third and fourth kinds of connection, because they will help you use the facts later.

Hint 2.
'Trying to learn' facts is completely useless.
If you make connections without trying to learn, you learn.
If you try to learn but don't make connections, you don't learn.

Hint 3.
The lectures are full of facts.
But most of these facts are not for learning.
Never try to memorise whole sections of lecture notes.

Instead look for the facts that you need to remember.
The lecturer will point out some of these.
You should remember *definitions* and *theorems*.
Don't learn *examples* unless the lecturer says you should.
They have a different purpose (read on).

Learning a skill

There are two stages in learning a skill.
The *first stage* is to have the rules in front of you and follow them.
Examples in the lectures are meant to help you with this.
The *second stage* is where you can do the task automatically, without having to refer back to the rules for each step.

You should always be aiming for the *second stage*.
(In fact it uses different parts of your brain from the first stage.)
Human beings have only one way of getting from the first stage to the second, namely

Hint 4.

Do lots of examples—not just the ones that the lecturer sets you.

Hint 5.
Watching somebody else do examples, and copying out other people's solutions, are completely useless for learning a skill.
At best they can tell you the rules, or show you where you were going wrong.
They will never get you anywhere near the second stage of learning.

Every skill builds on earlier, simpler ones.
If you can't learn a skill, it may be either because you can't do one of the steps involved,
or because you don't know how to fit the steps together.
It's up to you to work out exactly what your problem is.

Hint 6.
Identifying your difficulty is half way to solving it.

When you ask for advice, it should always be
'I can get this far, but then I'm stuck because . . . ',
or maybe
'When I follow the rules like this, I get the wrong answer'.

It should NEVER EVER be:
'I still can't do this kind of problem. Please do some more examples for me.'

I repeat:
It should NEVER EVER be:
'I still can't do this kind of problem. Please do some more examples for me.'

Learning proofs

Sometimes you will be asked in exams to reproduce proofs given in lectures.
This is an unfortunate byproduct of our system of written exams; it is neither good for you nor a reliable test of your abilities.
(The East European system of oral examinations doesn't have this problem, but it has others.)
To make the best of a bad job:

Hint 7.
Don't try to learn a proof word by word.
Instead try to learn its *shape*,
starting with the overall shape (e.g. the proof falls into three parts)
and gradually working down to the details.

This takes time. Don't leave it all to revision.

It's important that *you* find the shapes, though the lecturer should give hints.
As you get more experience, you should become better at noticing the shapes that matter.

Revision

You won't have much time for revision, so you need to use it wisely.
There is no time to learn new skills during revision. So cut your losses and do what can usefully be done in revision:

1. Memorise key facts and definitions. (You should already have identified which these are.)
2. Practise some skills which you *have* learned, to get them up to speed and reliable.
3. Read through the notes as a whole, looking for patterns. (E.g. when two definitions are similar, what are the differences? Which parts of the course depend on which other parts? As always, the patterns that will help you are the ones you find for yourself.)
4. Where you have a *precise* problem (not a vague 'I can't see what's going on'), ask the lecturer.
5. If absolutely necessary, memorise some key proofs *which you already understand*, and pray they will come up in the exams.