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Divisors of even numbers

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Abstract

The greatest common divisor of the even numbers is 2.

Key Words: greatest common divisor, even number

1. Introduction

It is well known that, while only 0 is divisible by 0, every number is divisible by 1. We show that every even number is divisible by 2, but by no greater number.

2. Preliminary results

Let us recall:

Definition 2.1 *A number is* ***even*** *if it is the sum of two equal numbers: Thus a is even, if* $a = b + c$*, where* $b = c$*.*

Lemma 2.2 *Even numbers are divisible by* 2*.*

Proof. Let a be an even number. Then, by Definition 2.1, $a = b + c$, where $b = c$. Thus $a = b + b = 1 \cdot b + 1 \cdot b = (1 + 1) \cdot b = 2 \cdot b$. □

3. The main theorem

Without further ado, let us proceed:

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Theorem 3.1 *The even numbers have a greatest common divisor, which is* 2.

Proof. We have already established in Lemma 2.2 that 2 divides every even number. Suppose $a > 2$. We must show that there is an even number b that is indivisible by a. Let $b = 2$. Then the remainder of b after division by a is b itself, namely 2. But $2 \neq 0$. Therefore a does not divide b. But b was an even number. Therefore not every even number is divisible by a. But a was an arbitrary number greater than 2. Therefore the theorem is proved. □

We hope to prove in a forthcoming paper [1] that the greatest common divisor of the square numbers is 1. This is believed to have been known to the ancients [2], but the proof has been lost.

References

[1] S. Beauchamp, Divisors of squares, preprint.

[2] Philomathus of Ancyra, *Thesaurus of Mathematics,* lost manuscript.

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