$F = \langle \sigma_v, \sigma_v \rangle = 0$

$G = \langle \sigma_w, \sigma_w \rangle = \frac{1}{w^4} + \frac{w^2-1}{w^4} = \frac{1}{w^2}$

$\boxed{K = -1}$

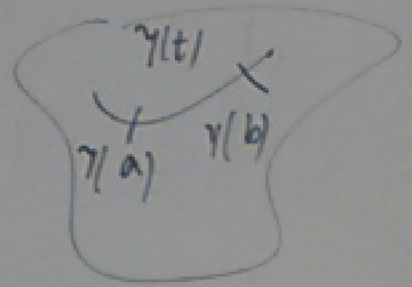
$(e^u, 0, \sqrt{1-e^{2u}} - \cosh^{-1}(e^{-u}))$

Riemannian Metrik

$U \subset \mathbb{R}^2$

$E(x,y)\, dx^2 + 2F(x,y)\, dx\, dy + G(x,y)\, dy^2$

Eğer $EG - F^2 \geq 0$ bu Riemannian Metrik denir.




$\gamma(t) = (a(t), b(t))$

$\dot{\gamma}(t) = (\dot{a}(t), \dot{b}(t))$

Bunun uzunluk $\int_a^b \|\dot{\gamma}(t)\|\, dt$

$= \int_a^b \left[ E(x,y)\, \dot{a}(t)^2 + 2F(x,y)\, \dot{a}(t)\, \dot{b}(t) + G(x,y)\, \dot{b}(t)^2 \right]^{1/2} dt$

$dx^2 + dy^2$ : Öklid Metrik

Örnek üst yarı düzlem


$H = \{ (x,y) \mid y > 0 \}$

Bu Metrik kullanacağız

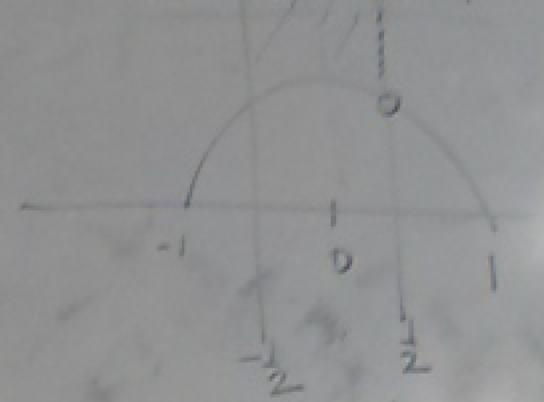
$\frac{dx^2 + dy^2}{y^2}$ (Hiperbolik Metrik)

$E = G = \frac{1}{y^2}$

$F = 0$

Soru: Bunun Gaussian Eğriliği nedir?

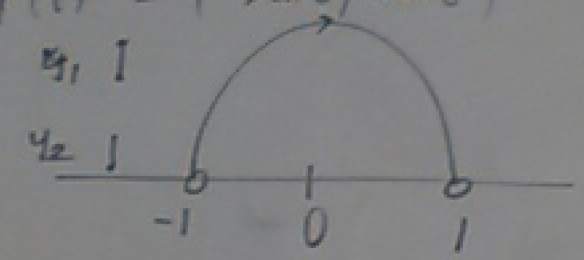
Bölge




5)

$\gamma(t) = (\cos t, \sin t) \quad -\pi < t < 0$

$\dot\gamma(t) = (-\sin t, \cos t)$




Bu yarı çemberin hiperbolik metriğ'e göre uzunluk nedir?

$\quad a(t) \quad b(t)$

$\dot\gamma(t) = (-\sin t, \cos t)$

$\dot a(t)^2 \times E + 2F\,\dot a(t)\dot b(t) + G\,\dot b(t)^2 \to \cos^2 t$

$\sin^2 t \qquad E = \frac{1}{\sin^2 t} = \frac{1}{y^2} \qquad F = 0 \qquad G = \frac{1}{y^2} = \frac{1}{\sin^2 t}$

$$\text{Uzunluk} = \int_{-\pi}^{0} \frac{1}{|\sin t|}\left(\cos^2 t + \sin^2 t\right)^{1/2} dt$$

$$= \int_{-\pi}^{0} \frac{dt}{|\sin t|} = \int_{0}^{\pi} \frac{dt}{\sin t} = \infty$$

mesafe koruyan dönüşümler $\cong PSL_2(\mathbb{R})$

$SL_2(\mathbb{R}) \curvearrowright \mathbb{H}$

$\mathbb{H} \ni z \longrightarrow \frac{az+b}{cz+d}$

$SL_2(\mathbb{R}) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(\mathbb{R})$

$\det = 1$

$ad - bc = 1$

hiperbolic geometry $\sim PSL(2,\mathbb{R})$

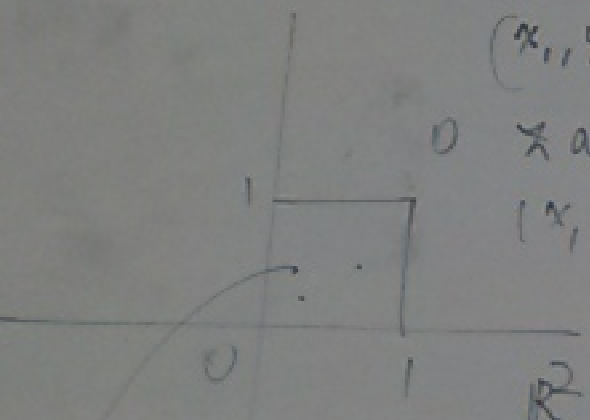
$F_2$

Öklid Geometri

$(x_1, y_1) = (x_2, y_2) + (m, n)$

o zaman

$(x_1, y_1) \sim (x_2, y_2)$



$\mathbb{R}^2 / \mathbb{Z}^2$

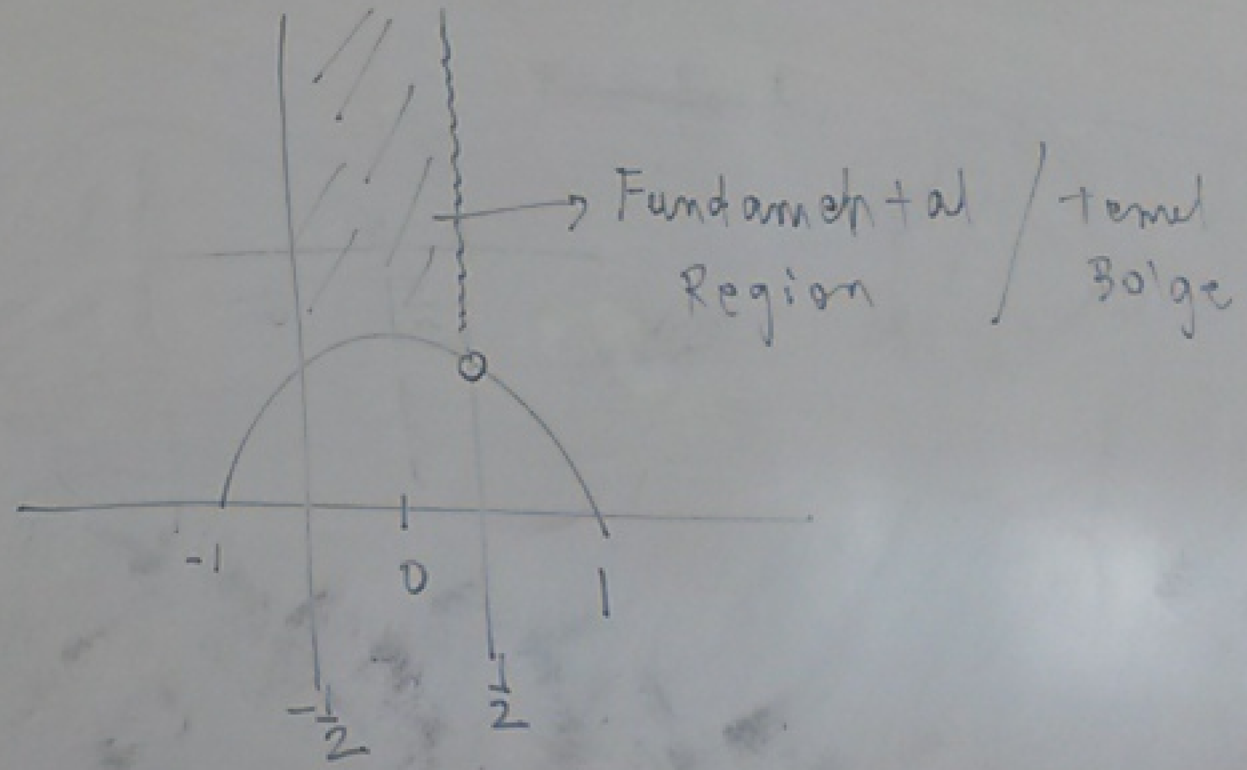
$[0,1) \times [0,1)$

→ Temel Bölge

Hiperbolik geometri

$z_1 \sim z_2$ eğer;

$z_1 = \dfrac{a z_2 + b}{c z_2 + d}$, $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2(\mathbb{Z})$ içinde bir matris için.



$\gamma(t) = (u(t), v(t))$ : Geo... olsun

$$\frac{d}{dt}\left(E\dot u + F\dot v\right) = \frac{1}{2}\left(E_u \dot u^2 + 2F_u \dot u\dot v + G_u \dot v^2\right) \longrightarrow \frac{d}{dt}\left(\frac{\dot u}{v^2}\right) = 0$$

$$\frac{d}{dt}\left(F\dot u + G\dot v\right) = \frac{1}{2}\left(E_v \dot u^2 + 2F_v \dot u\dot v + G_v \dot v^2\right) \longrightarrow \frac{d}{dt}\left(\frac{\dot v}{v^2}\right) = -\left(\frac{\dot u^2 + \dot v^2}{3}\right)$$

$$w = \frac{\sqrt{2}}{t}$$

$$k_g = \frac{\|a\|}{1-\|a\|^2}$$

$$k_n = \frac{-1}{r^2}$$

$$-\frac{\|a\|}{r^2} = k_g$$

$$\ddot\gamma = k_n \gamma + k_g\, \mathbf{n} \times \dot\gamma$$

$$(r\cos\tfrac{t}{r}, r\sin\tfrac{t}{r})$$

$$\ddot\gamma = -\frac{1}{r}\left(\cos\tfrac{t}{r}, \sin\tfrac{t}{r}\right) = -\frac{1}{r^2}\gamma$$

$$\boxed{\ddot\gamma = -\frac{1}{r}(\gamma - a)}$$



$$\sigma(v,w) = \left(\frac{1}{w}\cos v, \frac{1}{w}\sin v, \sqrt{1-\frac{1}{w^2}} - \cosh^{-1}(w)\right)$$

Birinci Temel formu nedir?

$$\sigma_v = \left(\frac{-\sin v}{w}, \frac{\cos v}{w}, 0\right)$$

$$\sigma_w = \left(-\frac{\cos v}{w^2}, -\frac{\sin v}{w^2}, \frac{1}{2}\left(1-\frac{1}{w^2}\right)^{-1/2}\frac{2}{w^3} - \frac{1}{\sqrt{w^2-1}}\right)$$

$$= \left(-\frac{\cos v}{w^2}, -\frac{\sin v}{w^2}, -\frac{\sqrt{w^2-1}}{w^2}\right)$$

$$E = \langle \sigma_v, \sigma_v\rangle = \frac{1}{w^2}$$

$$F = \langle \sigma_v, \sigma_w\rangle = 0$$

$$G = \langle \sigma_w, \sigma_w\rangle = \frac{1}{w^4} + \frac{w^2-1}{w^4} = \frac{1}{w^2}$$

Çevrilmiş yüzeyler

$$k = -\frac{\ddot f}{f}$$

$$\boxed{K = -1}$$

Birinci Temel formu:

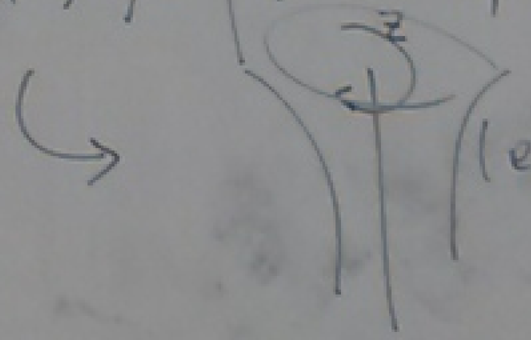
$$\frac{dv^2 + dw^2}{w^2}$$

Bu Hiperbolik Yarı Düzlem için bir model.

PSEUDOSPHERE (SAHTE KÜRE)

Bazen, $w = e^{-u}$ yazarak,

$$\tilde\sigma(u,v) = \left(e^{u}\cos v, e^{u}\sin v, \sqrt{1-e^{2u}} - \cosh^{-1}(e^{-u})\right)$$



$$\left(e^{u}, 0, \sqrt{1-e^{2u}} - \cosh^{-1}(e^{-u})\right)$$

$E = \langle \sigma_v, \sigma_v \rangle = \frac{1}{w^2}$

$F = \langle \sigma_v, \sigma_v \rangle = 0$

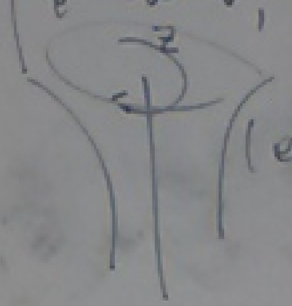
$G = \langle \sigma_w, \sigma_v \rangle = \frac{1}{w^4} + \frac{w^2-1}{w^4} = \frac{1}{w^2}$

Çevrilmiş yüzeyler

$\kappa = \frac{-\ddot{f}}{f}$

$\boxed{K = -1}$

$\sigma(u,v) = \left(e^u \cos v,\ e^u \sin v,\ \sqrt{1-e^{2u}} - \cosh^{-1}(e^{-u})\right)$



$\left(e^u, 0, \sqrt{1-e^{2u}} - \cosh^{-1}(e^{-u})\right)$

Not

$B(0,R) \xrightarrow{K} \frac{1}{R^2}$

Pseudosphere $\xrightarrow{K} -1$

Sanki yarıçapı $\sqrt{-1}$ olan bir küre.

$\sigma(u,v) = \left(\frac{\cos u}{v}, \frac{\sin u}{v}, \sqrt{1-\frac{1}{v^2}} - \cosh^{-1}(v)\right)$

$E = G = \frac{1}{v^2}$

$F = 0$

$\gamma(t) = (u(t), v(t))$ : Geodezik olsun

$\frac{d}{dt}(E\dot{u} + F\dot{v}) = \frac{1}{2}(E_u \dot{u}^2 + 2F_u \dot{u}\dot{v} + G_u \dot{v}^2) \longrightarrow \frac{d}{dt}\left(\frac{\dot{u}}{v^2}\right) = 0$

$\frac{d}{dt}(F\dot{u} + G\dot{v}) = \frac{1}{2}(E_v \dot{u}^2 + 2F_v \dot{u}\dot{v} + G_v \dot{v}^2) \longrightarrow \frac{d}{dt}\left(\frac{\dot{v}}{v^2}\right) = -\left(\frac{\dot{u}^2 + \dot{v}^2}{v^3}\right)$

$\gamma$: Birim hız olsun.

$\Rightarrow \dot{u}(t)^2 + \dot{v}(t)^2 = 1$

$\left(u, v, \ln \frac{\cos v}{\cos u}\right)$

$\dot{u} = c v^2$

$\frac{d}{dt}\left(\frac{\dot{v}}{v^2}\right) = \frac{-1}{v^3}$

$w = \frac{1}{v}$

$\frac{d^2}{dt^2}(w) = w^3$

$w = \frac{\sqrt{2}}{t}$

$\kappa_g = \frac{\|a\|}{1-\|a\|^2}$

$\kappa_n = \frac{-1}{r^2}$

$\frac{-\|a\|}{r^2} = \kappa_g$

$-\frac{1}{r^2}(\gamma - a) \quad \ddot{\gamma} = \kappa_n \gamma + \kappa_g\, N \dot{\gamma}$

⑤

Hatırlatma

Geodezik $\Rightarrow$ hız sabittir.

$\sqrt{1-r^2}$

$\left(r \cos\frac{t}{r}, r \sin\frac{t}{r}\right)$

$\dot{\gamma} = -\frac{1}{r}\left(\cos\frac{t}{r}, \sin\frac{t}{r}\right)$

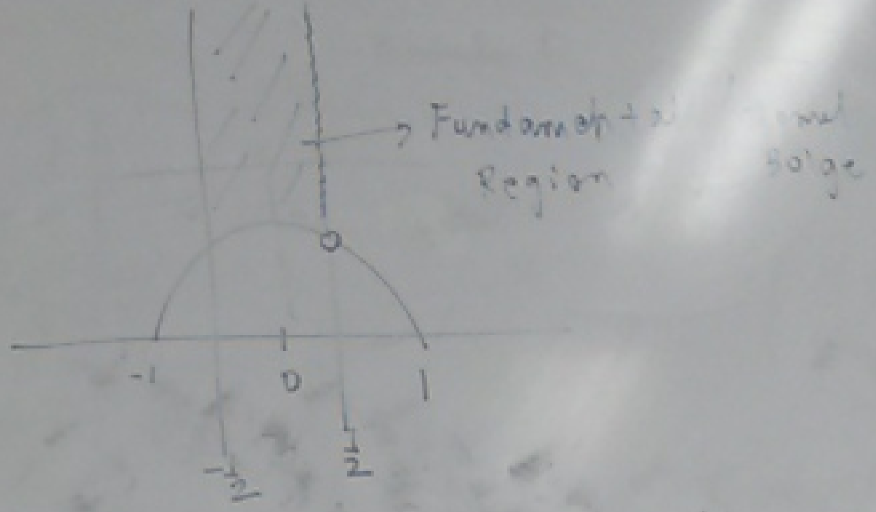
$= -\frac{1}{r^2}\gamma$

$\boxed{\ddot{\gamma} = -\frac{1}{r}(\gamma - a)}$

$\mathbb{R}^2/\mathbb{Z}^2$

$[0,1) \times [0,1)$

→ Temel Bölge



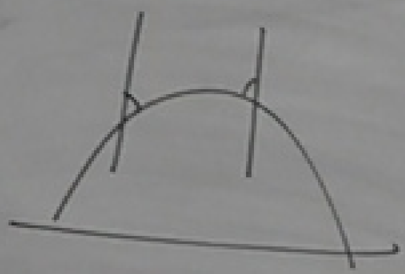

⑥

İddia

$y\ \alpha$

$(u(t)-\alpha)^2 + v(t)^2 = \text{sabit}$

Ödev

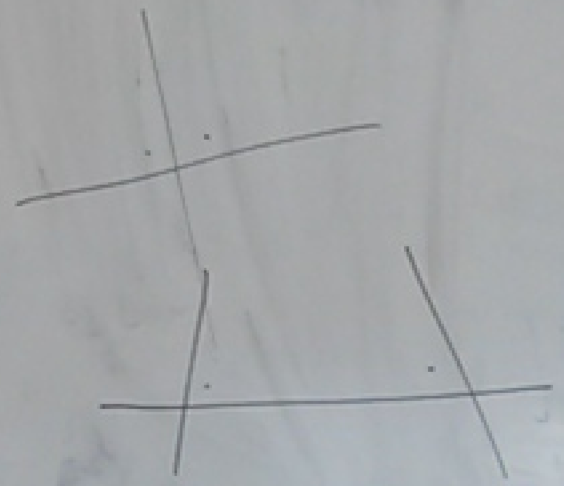


$(\cos t, \sin t)$

Hiperbolik geometri

Doğrular → x ekseni ile dik doğrular ve Merkezi x-eksen üzerinde olan yarı çemberler.

$EG - F^2 > 0$



Euclidean geometry ~ $\mathbb{Z}^2$

hiperbolic geometry ~ $PSL(2,\mathbb{Z})$

$F_2$

b) $\langle \sigma_u - \bar{N}_u, \sigma_v \rangle = F + M$
c) $\langle \sigma_v - \bar{N}_v, \sigma_u \rangle = F + M$
d) $\langle \sigma_v - \bar{N}_v, \sigma_v \rangle = G + N$

yüzey için 0.

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örnekler

(1)

**Sınav Soru 4**

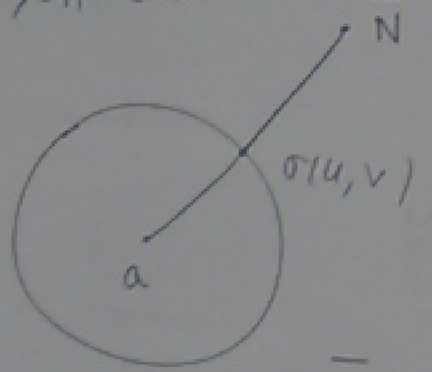
(S)

Eğer bir yüzeyin birinci ve ikinci temel formları

$$\cos^2 v\, du^2 + dv^2 \quad \text{ve} \quad -\cos^2 v\, du^2 - dv^2$$

ise, S bir yarıçapı 1 olan küre içindedir.

**Cevap**




Küre için $\sigma(u,v) - \bar{N} \equiv a$
bir a için

$\langle \bar{N}, \bar{N} \rangle \equiv 1$
$\Rightarrow \langle \bar{N}_u, \bar{N} \rangle \equiv 0$

Ayrıca $\sigma(u,v) - \bar{N} = a$ ise (sabit bir a için)
$\sigma(u,v) - a = \bar{N}$
$\Rightarrow \|\sigma(u,v) - a\| = 1$
$\Rightarrow$ S merkezi a olan küre içinde.

Demek ki bunu kanıtlamak lazım,

$\sigma_u - \bar{N}_u \equiv 0$
ve $\sigma_v - \bar{N}_v \equiv 0$

ancak ve ancak

$\langle \sigma_u - \bar{N}_u, \sigma_u \rangle \equiv 0 \quad \langle \sigma_u - \bar{N}_u, \sigma_v \rangle \equiv 0$
$\langle \sigma_v - \bar{N}_v, \sigma_u \rangle \equiv 0 \quad \langle \sigma_v - \bar{N}_v, \sigma_v \rangle \equiv 0$

Çünkü, $\sigma_v, \sigma_u \perp \bar{N}$
$\bar{N}_u, \bar{N}_v \perp \bar{N}$

$\Rightarrow$ Geçerli ise,
$\sigma_u - \bar{N}_u, \sigma_v - \bar{N}_v \perp \sigma_u, \sigma_v, \bar{N}$
Ama $\{\sigma_u, \sigma_v, \bar{N}\}$ bir bazdır.
$\Rightarrow \sigma_u - \bar{N}_u, \sigma_v - \bar{N}_v \equiv 0$

$\Rightarrow \sigma_u - \bar{N}_u , \sigma_v - \bar{N}_v \equiv 0$

<u>Not</u>

$E = \langle \sigma_u, \sigma_u \rangle \qquad L = \langle \sigma_{uu}, \bar{N} \rangle = -\langle \sigma_u, \bar{N}_u \rangle$

$F = \langle \sigma_u, \sigma_v \rangle \qquad M = \langle \sigma_{uv}, \bar{N} \rangle = -\langle \sigma_u, \bar{N}_v \rangle = -\langle \sigma_v, \bar{N}_u \rangle$

$G = \langle \sigma_v, \sigma_v \rangle \qquad N = \langle \sigma_{vv}, \bar{N} \rangle = -\langle \sigma_v, \bar{N}_v \rangle$

<u>Not</u> $\langle \sigma_u, \bar{N} \rangle \equiv 0$

$\Rightarrow \langle \sigma_{uu}, \bar{N} \rangle + \langle \sigma_u, \bar{N}_u \rangle = 0$

a) $\langle \sigma_u - \bar{N}_u, \sigma_u \rangle = E + L$

b) $\langle \sigma_u - \bar{N}_u, \sigma_v \rangle = F + M$

c) $\langle \sigma_v - \bar{N}_v, \sigma_u \rangle = F + M$

d) $\langle \sigma_v - \bar{N}_v, \sigma_v \rangle = G + N$

Bizim yüzey için 0.

②

<u>Not</u> eğer birinci ve ikinci temel form'un toplamı 0 ise, yüzey yarıçapı 1 olan küre içindedir.

Benzer şekilde,

$F_I$ : Birinci Temel formu

$F_{II}$ : Ikinci Temel formu

$F_I + R F_{II} \equiv 0 \iff S \hookrightarrow$ Yarıçapı R olan küre.

$$M_v - N_u = L\Gamma^1_{22} + M(\Gamma^2_{22} - \Gamma^1_{12}) - N\Gamma^2_{12}$$

(3)

**Tartışma** $E\,du^2 + 2F\,du\,dv + G\,dv^2$

$L\,du^2 + 2M\,du\,dv + N\,dv^2$

Bu katsayılar $E, F, G, L, M, N$ hakkında ne diyebiliriz. Bunlar tamamen serbest mi?

**Not** $E \geq 0$ çünkü $E = \langle \sigma_u, \sigma_u \rangle$

Aynı $G \geq 0$

$$\|\sigma_u \times \sigma_v\|^2 = (\sigma_u \cdot \sigma_u)(\sigma_v \cdot \sigma_v) - (\sigma_u \cdot \sigma_v)^2 = EG - F^2$$

$\Rightarrow EG - F^2 \geq 0$.

**Not** Birinci Temel formu için sadece bu 3 şart var $E \geq 0$ $G \geq 0$ $EG - F^2 \geq 0$.

**Soru** $F_I = du^2 + \cos^2 u\, dv^2$

$F_{II} = \cos^2 u\, du^2 + dv^2$

birinci ve ikinci temel formlar bu olan hiç bir yüzey yok.

**Hatırlatma**

$L = \langle \sigma_{uu}, \bar{N} \rangle = -\langle \sigma_u, \bar{N}_u \rangle$

$\mathcal{G} : S \longrightarrow S^2 \qquad \sigma(u,v) \longrightarrow \bar{N}_{\sigma(u,v)}$

$\mathcal{W} : T_pS \longrightarrow T_pS \qquad \mathcal{W} = -\mathcal{G}'$

**Not** $\mathcal{W}(\sigma_u) = -\bar{N}_u$

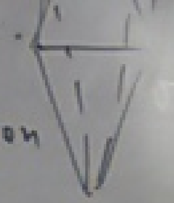
$\mathcal{W}(\sigma_v) = -\bar{N}_v$

$L = \langle \mathcal{W}(\sigma_u), \sigma_u \rangle$

$M = \langle \mathcal{W}(\sigma_u), \sigma_v \rangle = \langle \mathcal{W}(\sigma_v), \sigma_u \rangle$

$N = \langle \mathcal{W}(\sigma_v), \sigma_v \rangle$

$(k, l) = (3, 5) \Rightarrow$ $\left.\begin{array}{l} E = 30 \\ V = 20 \\ F = 12 \end{array}\right\}$ Icosahedron



$\sigma_v(0,0) = (0,1,0)$

$N(0,0) = (0,0,1)$



**Not** $W$ simetrik bir matristir.

$\Leftrightarrow (\langle W\xi, \eta\rangle = \langle \xi, W\eta\rangle)$

$\Leftrightarrow \left(W \sim \begin{bmatrix} \alpha & \beta \\ \beta & \gamma \end{bmatrix}\right.$ bir onb olarak yazarsak)

**Hatırlatma**

$F_I$ ve $F_{II}$ (Birinci ve İkinci Temel formlar tamamen başımsız değil)

İki ilişki var.

Codazzi - Mainardi Denklemleri

$$L_v - M_u = L\Gamma^1_{12} + M(\Gamma^2_{12} - \Gamma^1_{11}) - N\Gamma^2_{11}$$

$$M_v - N_u = L\Gamma^1_{22} + M(\Gamma^2_{22} - \Gamma^1_{12}) - N\Gamma^2_{12}$$

$\Gamma^i_{jk}$ : Christoffel semboller.

**Teorem** $F_I = E\,du^2 + 2F\,du\,dv + G\,dv^2$

$F_{II} = L\,du^2 + 2M\,du\,dv + N\,dv^2$

Varsayalım

a) $E, G \geq 0 \quad EG - F^2 > 0$

b) Codazzi - Mainardi denklemler sağlasın.

O zaman birinci ve ikinci temel formlar $F_I$ ve $F_{II}$ olan bir yüzey var.

$\frac{2}{k} + \frac{2}{l} = \frac{}{E}$

Eğer $\nearrow \geq 1$ $\begin{pmatrix} k \geq 3 \\ l \geq 3 \end{pmatrix}$

$(k,l) = (3,3), (3,4), (4,3)$
$(3,5), (5,3)$

örnekler

Hatırlatma Christoffel sembolleri

$\{\sigma_u, \sigma_v, \bar{N}\}$ baz

$$\begin{bmatrix} \sigma_{uu} \\ \sigma_{uv} \\ \sigma_{vv} \end{bmatrix} = \begin{bmatrix} \Gamma^1_{11} & \Gamma^2_{11} & L \\ \Gamma^1_{12} & \Gamma^2_{12} & M \\ \Gamma^1_{22} & \Gamma^2_{22} & N \end{bmatrix} \begin{bmatrix} \sigma_u \\ \sigma_v \\ \bar{N} \end{bmatrix}$$

$\{\Gamma^i_{jk}\}$ sadece birinci temel formu ile yazılabilir.

Eğer BFT $F_I = du^2 + G\,dv^2$ ise

$\Gamma^1_{11} = 0$ $\quad \Gamma^2_{11} = 0$

$\Gamma^1_{12} = 0$ $\quad \Gamma^2_{12} = \frac{G_u}{2}$

$\Gamma^1_{22} = -\frac{G_u}{2}$ $\quad \Gamma^2_{22} = \frac{G_v}{2G}$

Bizim örnek : $du^2 + \cos^2 u\, dv^2$

$$\begin{bmatrix} 0 & 0 \\ 0 & -\cos u \sin u \\ \cos u \sin u & 0 \end{bmatrix} = \begin{bmatrix} \Gamma^1_{11} & \Gamma^2_{11} \\ \Gamma^1_{12} & \Gamma^2_{12} \\ \Gamma^1_{22} & \Gamma^2_{22} \end{bmatrix}$$

Codazzi-Minardi

6

$$I = du^2 + \cos^2 u \, dv^2$$

$$II = \cos^2 u \, du^2 + dv^2$$

$$\underset{\substack{\| \\ 0}}{L_v - M_u} \overset{?}{=} L\Gamma^1_{12} + \underset{\substack{\| \\ 0}}{M(\Gamma^2_{12} - \Gamma^1_{11})} - N\Gamma^2_{11}$$

$$\underset{\substack{\| \\ 0}}{M_v - N_u} \overset{?}{=} L\Gamma^1_{22} + M(\Gamma^2_{22} - \Gamma^1_{12}) - N\Gamma^2_{12}$$

$$\cos^3 u \sin u \qquad + \cos u \sin u$$

$$= \cos u \sin u (\cos^2 u + 1)$$

$$\neq$$

Her köşeden $k$ kenar çıkıyor

Her yüzeyin etrafında $l$ kenar var.

$$Vk = 2E$$
$$Fl = 2E$$

$V$: köşe sayısı

$E$: kenar "

$F$: yüzey "

$$V + F = E + 2 \quad \text{(Euler)}$$

$$\Rightarrow \frac{2E}{k} + \frac{2E}{l} = E + 2$$

$$\Rightarrow \frac{2}{k} + \frac{2}{l} = 1 + \frac{2}{E}$$

Özellikle, $\frac{2}{k} + \frac{2}{l} \geq 1$

Eğer $\geq 1$ $\left( \begin{array}{l} k \geq 3 \\ l \geq 3 \end{array} \right)$

$(k,l) = (3,3), (3,4), (4,3)$
$(3,5), (5,3)$

$K = \det(\ldots) = \ldots \frac{1}{2} \neq 0$

$(k,l) = (3,3) \Rightarrow \left.\begin{array}{l} E = 6 \\ V = 4 \\ F = 4 \end{array}\right\}$ Tetrahedron

$(k,l) = (3,4) \Rightarrow \left.\begin{array}{l} E = 12 \\ V = 8 \\ F = 6 \end{array}\right\}$ Cube

$(k,l) = (4,3) \Rightarrow \left.\begin{array}{l} E = 12 \\ F = 8 \\ V = 6 \end{array}\right\}$ Octahedron

$(k,l) = (3,5) \Rightarrow \left.\begin{array}{l} E = 30 \\ V = 20 \\ F = 12 \end{array}\right\}$ Icosahedron

$(k,l) = (5,3) \Rightarrow \left.\begin{array}{l} E = 30 \\ F = 20 \\ V = 12 \end{array}\right\}$ Dodecahedron

Soru 4 (7)

$$r(u,v) = \left( \left(1 - u\sin\frac{v}{2}\right)\cos v,\ \left(1 - u\sin\frac{v}{2}\right)\sin v,\ u\cos\frac{v}{2} \right)$$

$K \neq 0$

$r_u(0,0) = (0,0,1)$ $\qquad g_I(0,0) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$r_v(0,0) = (0,1,0)$

$N(0,0) = (0,0,1)$

$$r_u = \left( -\sin\frac{v}{2}\cos v,\ -\sin\frac{v}{2}\sin v,\ \cos\frac{v}{2} \right)$$

$$r_v = \left( -\sin v - \frac{u}{2}\cos\frac{v}{2}\cos v + u\sin\frac{v}{2}\sin v,\ \cos v - \frac{u}{2}\cos\frac{v}{2}\sin v - u\sin\frac{v}{2}\cos v,\ -\frac{u}{2}\sin\frac{v}{2} \right)$$

$(k, l) = (3,5) \Rightarrow$ $E = 30$, $V = 20$, $F = 12$ } Icosahedron

$(k, l) = (5, 3) \Rightarrow$ $E = 30$, $F = 20$ } Dodecahedron

$\bar{N}(0,0) = (0,0,1)$

$\sigma_u = \left(-\sin\frac{v}{2}\cos v, -\sin\frac{v}{2}\sin v, \cos\frac{v}{2}\right)$

$\sigma_v = \left(-\sin v - \frac{u}{2}\cos\frac{v}{2}\cos v + u\sin\frac{v}{2}\sin v, \cos v - \frac{u}{2}\cos\frac{v}{2}\sin v \ldots\right.$

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$$\sigma_{uu}(0,0) = 0$$

$$\sigma_{uv}(0,0) = \frac{d}{dt}\sigma_u(0,t)\Big|_{t=0} = \left(-\frac{1}{2}, 0, 0\right)$$

$$\sigma_{vv}(0,0) = \frac{d}{dt}\sigma_v(0,t) = (-1, 0, 0)$$

$$F_{II}(0,0) = \begin{bmatrix} 0 & -1/2 \\ -1/2 & -1 \end{bmatrix}$$

$$W = F_I^{-1} F_{II} = \begin{bmatrix} 0 & -1/2 \\ -1/2 & -1 \end{bmatrix}$$

$$K = \det(W) = -\frac{1}{4} \neq 0$$

$\sigma(0,0)$ noktada $K$ sıfır değil.

$\Rightarrow$ Theorema Egregium'e göre bunu düzlem parçaları kesmeden, inşa etmek mümkün değildir.