$F = \langle \sigma_v, \sigma_v \rangle = 0$

$G = \langle \sigma_w, \sigma_w \rangle = \frac{1}{w^4} + \frac{w^2-1}{w^4} = \frac{1}{w^2}$

$\boxed{K = -1}$

$(e^u, 0, \sqrt{1-e^{2u}} - \cosh^{-1}(e^{-u}))$

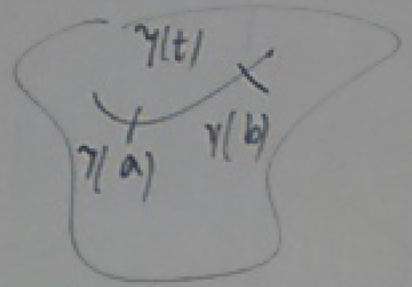
1

Riemannian Metrik

$U \subset \mathbb{R}^2$

$E(x,y)\,dx^2 + 2F(x,y)\,dx\,dy + G(x,y)\,dy^2$

Eğer $EG - F^2 \geq 0$ bu Riemannian metrik denir.



Bunun uzunluk $\int_a^b \|\dot{\gamma}(t)\|\,dt$

$\gamma(t) = (a(t), b(t))$

$\dot{\gamma}(t) = (\dot{a}(t), \dot{b}(t))$

$$= \int_a^b \left[E(x,y)\,\dot{a}(t)^2 + 2F(x,y)\,\dot{a}(t)\,\dot{b}(t) + G(x,y)\,\dot{b}(t)^2 \right]^{1/2} dt$$

$\boxed{dx^2 + dy^2 : \text{Öklid Metrik}}$

Örnek üst yarı düzlem

$H = \{(x,y) \mid y > 0\}$

Bu metrik kullanacağız

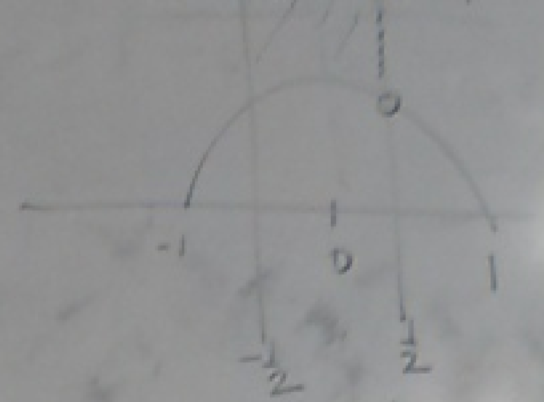
$$\frac{dx^2 + dy^2}{y^2} \quad \text{(Hiperbolik Metrik)}$$

$E = G = \frac{1}{y^2}$

$F = 0$

Soru: Bunun Gaussian Eğriliği nedir?

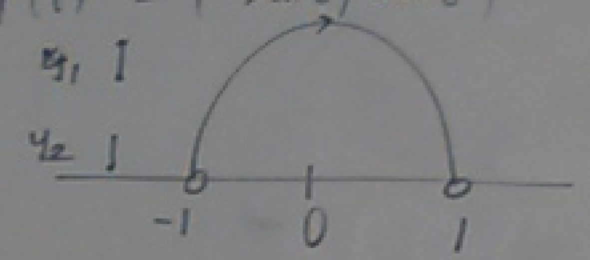
Bölge



(6)

$\gamma(t) = (\cos t, \sin t) \qquad -\pi < t < 0$

$\dot\gamma(t) = (-\sin t, \cos t)$



Bu yarı çemberin hiperbolik metriğe göre uzunluk nedir?

$$\dot\gamma(t) = (\underbrace{-\sin t}_{\dot a(t)}, \underbrace{\cos t}_{\dot b(t)})$$

$$\dot a(t)^2 \times E + 2F\,\dot a(t)\dot b(t) + G\,\dot b(t)^2 \longrightarrow \cos^2 t$$

$$\sin^2 t \qquad \frac{1}{\sin^2 t} = \frac{1}{y^2} \qquad 0 \qquad \frac{1}{y^2} = \frac{1}{\sin^2 t}$$

$$\text{Uzunluk} = \int_{-\pi}^{0} \frac{1}{|\sin t|}\left(\cos^2 t + \sin^2 t\right)^{1/2} dt$$

$$= \int_{-\pi}^{0} \frac{dt}{|\sin t|} = \int_{0}^{\pi} \frac{dt}{\sin t} = \infty$$

Mesafe koruyan dönüşümler $\cong PSL_2(\mathbb{R})$

$SL_2(\mathbb{R}) \curvearrowright \mathbb{H}$

$\mathbb{H} \ni z \longrightarrow \dfrac{az+b}{cz+d}$

$SL_2(\mathbb{R}) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(\mathbb{R})$

$\det = 1$

$ad - bc = 1$

hiperbolic geometry ~ PSL(2, R)

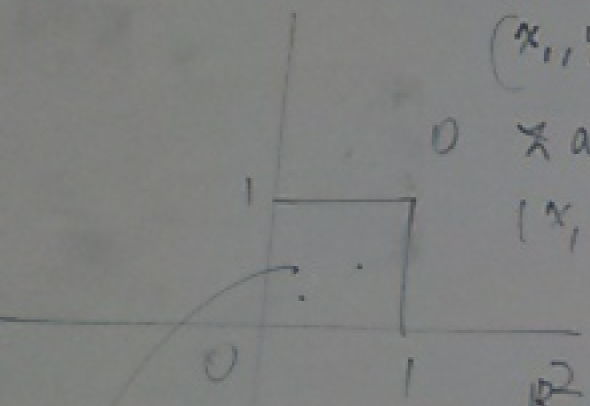
F_2

Öklid Geometri

$(x_1, y_1) = (x_2, y_2) + (m, n)$

o zaman

$(x_1, y_1) \sim (x_2, y_2)$



$\mathbb{R}^2 / \mathbb{Z}^2$

$[0,1) \times [0,1)$

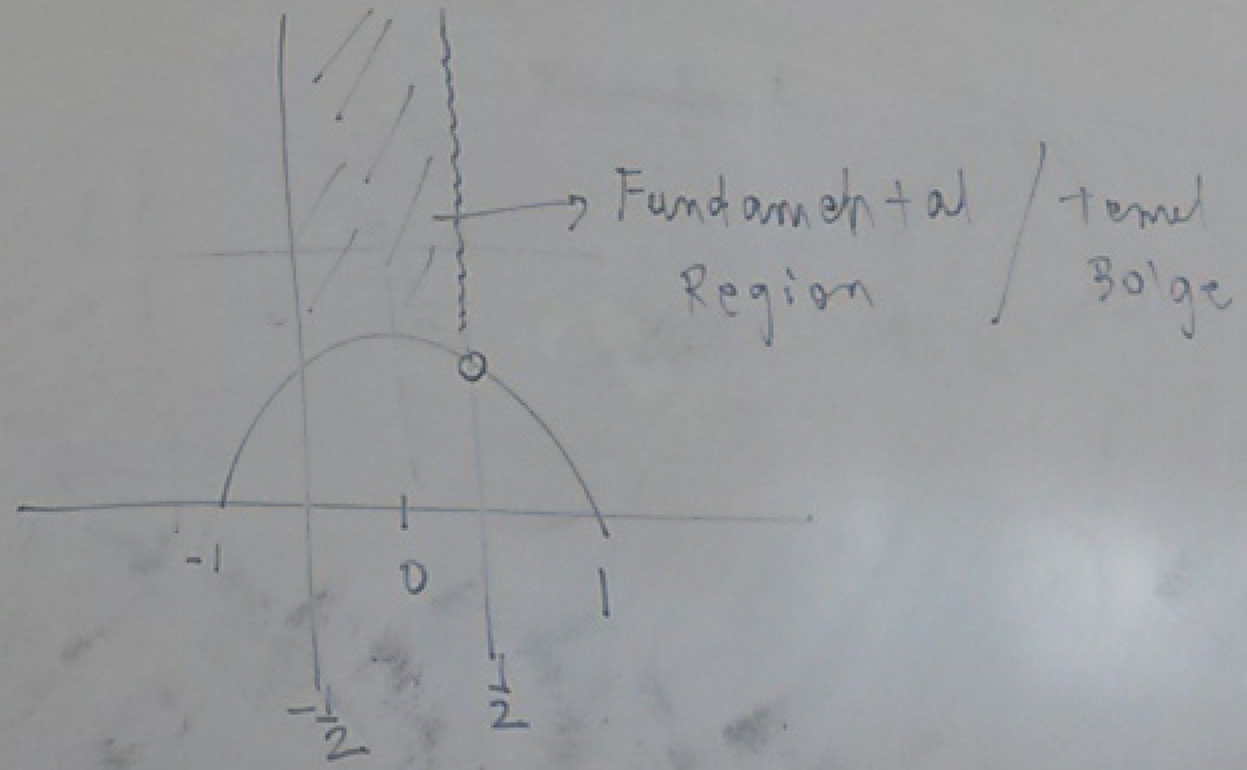
Temel Bölge

Hiperbolik geometri

$z_1 \sim z_2$ eğer,

$z_1 = \dfrac{az_2 + b}{cz_2 + d}$, $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2(\mathbb{Z})$ içinde

bir matris için.



$\gamma(t) = (u(t), v(t))$: Geodezik olsun

$$\frac{d}{dt}\left(E\dot u + F\dot v\right) = \frac{1}{2}\left(E_u \dot u^2 + 2F_u \dot u \dot v + G_u \dot v^2\right) \longrightarrow \frac{d}{dt}\left(\frac{\dot u}{v^2}\right) = 0$$

$$\frac{d}{dt}\left(F\dot u + G\dot v\right) = \frac{1}{2}\left(E_v \dot u^2 + 2F_v \dot u \dot v + G_v \dot v^2\right) \longrightarrow \frac{d}{dt}\left(\frac{\dot v}{v^2}\right) = -\left(\frac{\dot u^2 + \dot v^2}{3}\right)$$

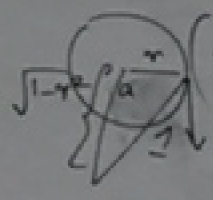
$w = \frac{\sqrt{2}}{t}$

$\kappa_g = \frac{\|a\|}{1-\|a\|^2}$

$\kappa_n = \frac{-1}{r^2}$

$-\frac{\|a\|}{r^2} = \kappa_g$

$-\frac{1}{r^2}(\gamma - a)$ $\ddot\gamma = \kappa_n \gamma + \kappa_g n \times \dot\gamma$



$\left(r\cos\frac{t}{r}, r\sin\frac{t}{r}\right)$

$\ddot\gamma = -\frac{1}{r}\left(\cos\frac{t}{r}, \sin\frac{t}{r}\right) = -\frac{1}{r^2}\gamma$

$\boxed{\ddot\gamma = -\frac{1}{r}(\gamma - a)}$

④

$$\sigma(v,w) = \left(\frac{1}{w}\cos v, \frac{1}{w}\sin v, \sqrt{1-\frac{1}{w^2}} - \cosh^{-1}(w)\right)$$

Birinci Temel formu nedir?

$$\sigma_v = \left(\frac{-\sin v}{w}, \frac{\cos v}{w}, 0\right)$$

$$\sigma_w = \left(-\frac{\cos v}{w^2}, -\frac{\sin v}{w^2}, \frac{1}{2}\left(1-\frac{1}{w^2}\right)^{-1/2}\frac{2}{w^3} - \frac{1}{\sqrt{w^2-1}}\right)$$

$$= \left(-\frac{\cos v}{w^2}, -\frac{\sin v}{w^2}, -\frac{\sqrt{w^2-1}}{w^2}\right)$$

$E = \langle \sigma_v, \sigma_v\rangle = \frac{1}{w^2}$

$F = \langle \sigma_v, \sigma_w\rangle = 0$

$G = \langle \sigma_w, \sigma_w\rangle = \frac{1}{w^4} + \frac{w^2-1}{w^4} = \frac{1}{w^2}$

Çevrilmiş yüzeyler

$\kappa = -\frac{\ddot f}{f}$

$\boxed{K = -1}$

Birinci Temel formu:

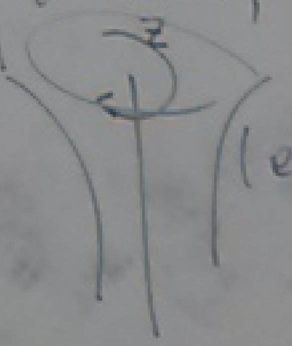
$$\frac{dv^2 + dw^2}{w^2}$$

Bu Hiperbolik Yarı Düzlem için bir model.

PSEUDOSPHERE (SAHTE KÜRE)

Bazen, $w = e^{-u}$ yazarak,

$$\tilde\sigma(u,v) = \left(e^u\cos v, e^u\sin v, \sqrt{1-e^{2u}} - \cosh^{-1}(e^{-u})\right)$$



$$\left(e^u, 0, \sqrt{1-e^{2u}} - \cosh^{-1}(e^{-u})\right)$$

$E = \langle \sigma_v, \sigma_v \rangle = \frac{1}{w^2}$

$F = \langle \sigma_v, \sigma_v \rangle = 0$

$G = \langle \sigma_w, \sigma_v \rangle = \frac{1}{w^4} + \frac{w^2-1}{w^4} = \frac{1}{w^2}$

Çevrilmiş yüzeyler

$k = \frac{-\ddot{f}}{f}$

$\boxed{K = -1}$

$\sigma(u,v) = \left(e^u \cos v, e^u \sin v, \sqrt{1-e^{2u}} - \cosh^{-1}(e^{-u}) \right)$

$\left(e^u, 0, \sqrt{1-e^{2u}} - \cosh^{-1}(e^{-u}) \right)$

Not

$B(0,R) \xrightarrow{K} \frac{1}{R^2}$

Pseudosphere $\xrightarrow{K} -1$

Sanki yarıçapı $\sqrt{-1}$ olan bir küre.

$\sigma(u,v) = \left(\frac{\cos u}{v}, \frac{\sin u}{v}, \sqrt{1-\frac{1}{v^2}} - \cosh^{-1}(v) \right)$

$E = G = \frac{1}{v^2}$

$F = 0$

$\gamma(t) = (u(t), v(t))$: Geodezik olsun

$\frac{d}{dt}(E\dot{u} + F\dot{v}) = \frac{1}{2}(E_u \dot{u}^2 + 2F_u \dot{u}\dot{v} + G_u \dot{v}^2) \longrightarrow \frac{d}{dt}\left(\frac{\dot{u}}{v^2}\right) = 0$

$\frac{d}{dt}(F\dot{u} + G\dot{v}) = \frac{1}{2}(E_v \dot{u}^2 + 2F_v \dot{u}\dot{v} + G_v \dot{v}^2) \longrightarrow \frac{d}{dt}\left(\frac{\dot{v}}{v^2}\right) = -\left(\frac{\dot{u}^2 + \dot{v}^2}{v^3}\right)$

γ: Birim hız olsun.

$\Rightarrow \dot{u}(t)^2 + \dot{v}(t)^2 = 1$

$\left(u, v, \ln \frac{\cos v}{\cos u}\right)$

(5)

$\dot{u} = c v^2$

$\frac{d}{dt}\left(\frac{\dot{v}}{v^2}\right) = \frac{-1}{v^3}$

$w = \frac{1}{v}$

$\frac{d^2}{dt^2}(w) = w^3$

$w = \frac{\sqrt{2}}{t}$

Hatırlatma

Geodezik $\Rightarrow$ hız sabittir.

$\kappa_g = \frac{\|a\|}{1 - \|a\|^2}$

$\kappa_n = \frac{-1}{r^2}$

$-\frac{\|a\|}{r^2} = \kappa_g$

$-\frac{1}{r^2}(\gamma - a)$ $\ddot{\gamma} = \kappa_n \gamma + \kappa_g N \dot{\gamma}$

$\sqrt{1-r^2}$

$\left(r \cos \frac{t}{r}, r \sin \frac{t}{r} \right)$

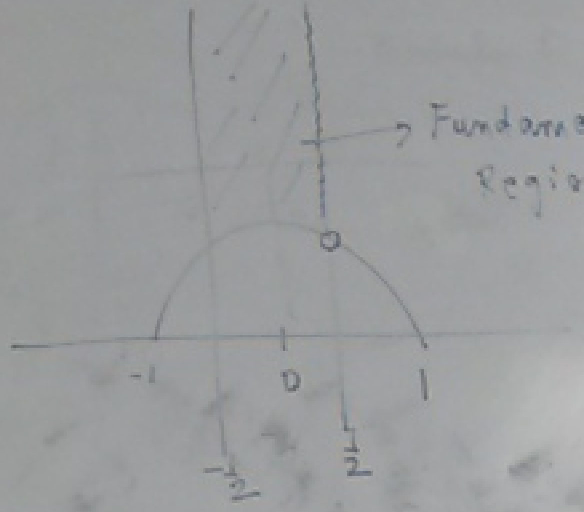
$\ddot{\gamma} = -\frac{1}{r}\left(\cos\frac{t}{r}, \sin\frac{t}{r}\right) = -\frac{1}{r^2}\gamma$

$\boxed{\ddot{\gamma} = -\frac{1}{r}(\gamma - a)}$

$\mathbb{R}^2 / \mathbb{Z}^2$

$[0,1) \times [0,1)$

→ Temel Bölge

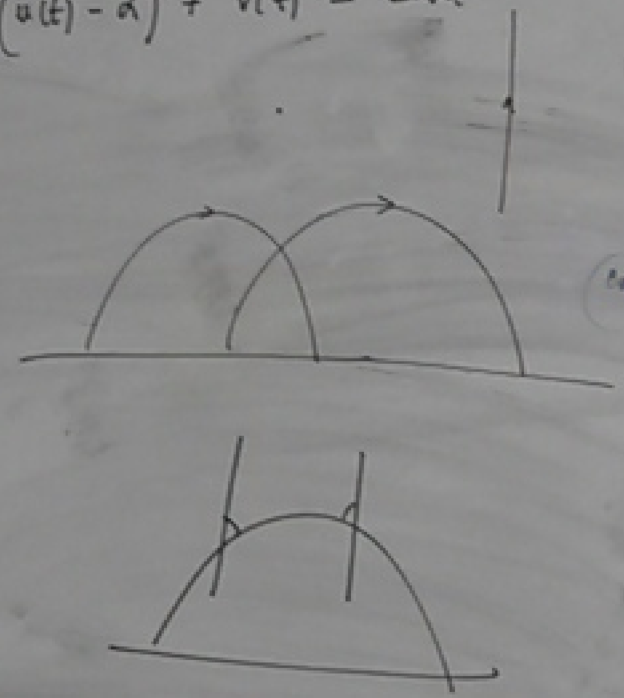


İddia

$y\ \alpha$

$(u(t) - \alpha)^2 + v(t)^2 = \text{sabit}$

Ödev

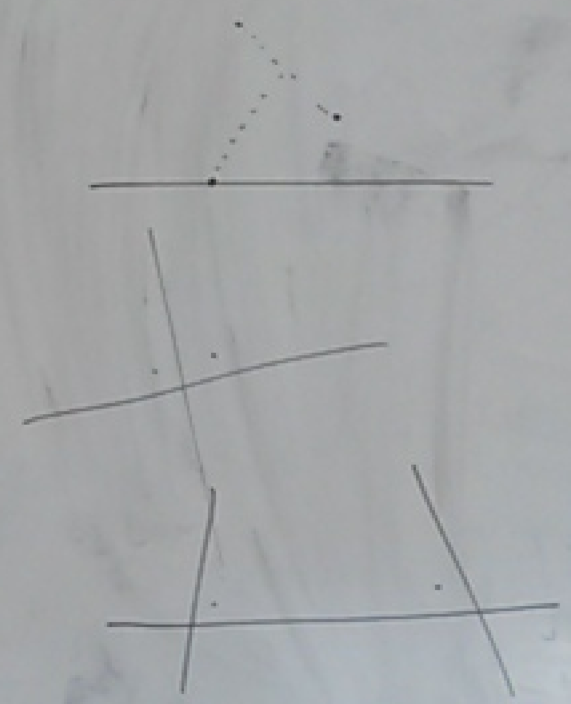


Hiperbolik geometri

Doğrular → x ekseni ile dik doğrular ve Merkezi x-ekseni üzerinde olan yarı çemberler.

(cos t, sin t, t)

$EG - F^2 > 0$



Euclidean geometry ~ $\mathbb{R}^2$

hiperbolic geometry ~ $PSL(2, \mathbb{R})$

F_2