VECTOR-SPACES OVER UNSPECIFIED FIELDS

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In the *Elements,* Euclid of Alexandria gives a geometric formulation of certain field-theoretic identities; for example, the identity

$$x^2 - y^2 = (x + y)(x - y)$$

is Euclid's Proposition II.5, but Euclid expresses in terms of the squares and rectangles bounded by certain lines. At the beginning of the *Geometry,* René Descartes shows how lines can be multiplied to produce *lines* rather than rectangles, if one line is chosen as unit. The construction justifies Descartes's algebraic treatment of geometry.

A model-theoretic version of Descartes's construction is that the theory of vector-spaces (as one-sorted structures, but over unspecified fields) can be axiomatized in the signature of abelian groups with a predicate for *parallelism* (binary linear dependence). The existentially closed models of this theory are two-dimensional spaces (over algebraically closed fields). If a predicate for n-ary linear dependence is introduced to the language, then the existentially closed models of the expanded theory are n-dimensional. In particular, a vector-space of dimension greater than n embeds in a space of dimension n so as to preserve independence in all n-element sets of vectors. This might be compared with the observation that existentially closed field-extensions have transcendence-degree one.

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