Basic Model-Theory (Math 736)

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Fall 2001

Text: There is no official text, besides the lectures and some notes I will provide on some topics; but most of the material can be found in the first half of [3]. Other references include the more recent [2], and the older [1].

About the subject: A **structure** is a set, possibly equipped with some distinguished functions and relations. Examples include groups, rings, and linear orders. **Model-theory** is the study of structures, as such or as **models** of particular **theories**. (In this context, a group is just a model of group-theory, in a technical sense of the word 'theory'—a sense which is *not* intended in the term 'model-theory' itself.)

Model-theory has been called 'algebraic geometry without fields.' If algebraic geometry is about polynomial equations over fields, then model-theory is about analogous formulas over arbitrary structures.

Model-theory has also been called 'the geography of tame mathematics.' The notions of 'tame' and its opposite, 'wild', are not precisely defined; but the structure $(\mathbf{N}, +, \cdot)$ of the natural numbers is wild (by Gödel's Incompleteness Theorem), while the structure $(\mathbf{C}, +, \cdot)$ of the complex numbers is tame for various reasons, which model-theory identifies and looks for in other structures as well.

About the course: The first theorem will be Compactness (the model-theoretic version of Gödel's Completeness Theorem). This is a model-existence result, saying for example that the theory of finite fields has infinite models. We shall define and examine—with motivating examples—*theories* that are: complete, model-complete, quantifier-eliminable, and categorical; and *structures* that are: prime, minimal, universal, saturated, and stable.

Prerequisites: No specific background is required, just some familiarity with some part of mathematics or logic.

References

[1] C. C. Chang and H. J. Keisler. *Model theory.* North-Holland Publishing Co., Amsterdam, third edition, 1990.

[2] Wilfrid Hodges. *Model Theory.* Cambridge University Press, 1993.

[3] Bruno Poizat. *A course in model theory.* Springer-Verlag, New York, 2000.