First-order logic exercises

Math 406

2004.11.02

Problem 1. *Letting P and Q be unary predicates, determine, from the definition of $\models$, whether the following hold:*

(a) $\exists x\ Px \to \exists x\ Qx \models \forall x\ (Px \to Qx)$;

(b) $\forall x\ Px \to \exists x\ Qx \models \exists x\ (Px \to Qx)$;

(c) $\exists x\ (Px \to Qx) \models \forall x\ Px \to \exists x\ Qx$.

Problem 2. *Let $\mathcal{L} = \{R\}$, where R is a binary predicate, and let $\mathfrak{A}$ be the $\mathcal{L}$-structure $(\mathbb{Z}, \leqslant)$. Determine $\phi^{\mathfrak{A}}$ if ϕ is:*

(a) $\forall x_1\ (Rx_1x_0 \to Rx_0x_1)$;

(b) $\forall x_2\ (Rx_2x_0 \lor Rx_1x_2)$.

Problem 3. *Let $\mathcal{L}$ be $\{S, P\}$, where S and P are binary function-symbols. Then $(\mathbb{R}, +, \cdot)$ is an $\mathcal{L}$-structure. Show that the following sets and relations are definable in this structure:*

(a) $\{0\}$;

(b) $\{1\}$;

(c) $\{a \in \mathbb{R} : 0 < a\}$;

(d) $\{(a, b) \in \mathbb{R}^2 : a < b\}$.

Problem 4. *Show that the following sets are definable in $(\omega, +, \cdot, \leqslant, 0, 1)$:*

(a) *the set of even numbers;*

(b) *the set of prime numbers.*