Name:

1	
2	
3	
4	
$\sum$	

METU MATH 365, EXAM 2

Thursday, December 16, 2010, at 17:40
Instructor: David Pierce

Instructions: There are 4 numbered problems (with a total of 8 parts) on 4 pages. Please work carefully. It should be obvious how to read your solutions.

Problem 1. Exactly one of 1458 and 1536 has a primitive root. Which one, and why? Find a primitive root of the number that has one.

Problem 2. Remembering that p is always prime, define the arithmetic function ω by

$$\omega(n) = \sum_{p|n} 1.$$

a. Define μ (the Möbius function), preferably using ω.

b. Prove that, if m and n are co-prime, then $\omega(mn) = \omega(m) + \omega(n)$.

c. Prove that

$$\sum_{d|n} \tau(d) \cdot \mu(d) = (-1)^{\omega(n)}.$$

d. Find a simple description of the function f given by

$$f(n) = \sum_{d|n} \omega(d) \cdot \mu\Big(\frac{n}{d}\Big).$$

Problem 3. Find the least positive x such that

$$11^{5117}x \equiv 5 \pmod{600}.$$

Problem 4.

a. Since 2 is a primitive root of 29, the function $x \mapsto \log_2 x$ from $\mathbb{Z}_{29}{}^{\times}$ to $\mathbb{Z}_{28}$ is defined. Considering this as a function from the set $\{-14, \ldots, -1, 1, \ldots 14\}$ to $\{-14, \ldots, 14\}$, fill out a table like the one below.

m	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\log_2 m$														
$\log_2(-m)$														

b. With respect to the modulus 29, exactly one of the two congruences

$$x^{400} \equiv 13, \qquad\qquad x^{400} \equiv -13$$

has a solution. Find all of its solutions (*modulo* 29), and explain why the other congruence has no solutions.