NUMBER-THEORY EXERCISES, III

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Here p and p_i are always prime numbers.

Exercise 1. $p \equiv \pm 1 \pmod 6$.

Exercise 2. If $p \equiv 1 \pmod 3$ then $p \equiv 1 \pmod 6$.

Exercise 3. If $n \equiv 2 \pmod 3$, then n has a factor p such that $p \equiv 2 \pmod 3$.

Exercise 4. Find all primes of the form $n^3 - 1$.

Exercise 5. Find all p such that $3p + 1$ is square.

Exercise 6. Find all p such that $p^2 + 2$ is prime.

Exercise 7. $n^4 + 4$ is composite.

Exercise 8. If n is positive, then $8^n + 1$ is composite.

Exercise 9. Find all integers n such that the equation

$$x^2 = ny^2$$

has only the zero solution. Prove your findings.

Exercise 10. If $p_0 < \cdots < p_n$, prove that the sum

$$\frac{1}{p_0} + \cdots + \frac{1}{p_n}$$

is not an integer.

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