MATH 304 EXAMINATION, TUESDAY, MAY 18, 2010

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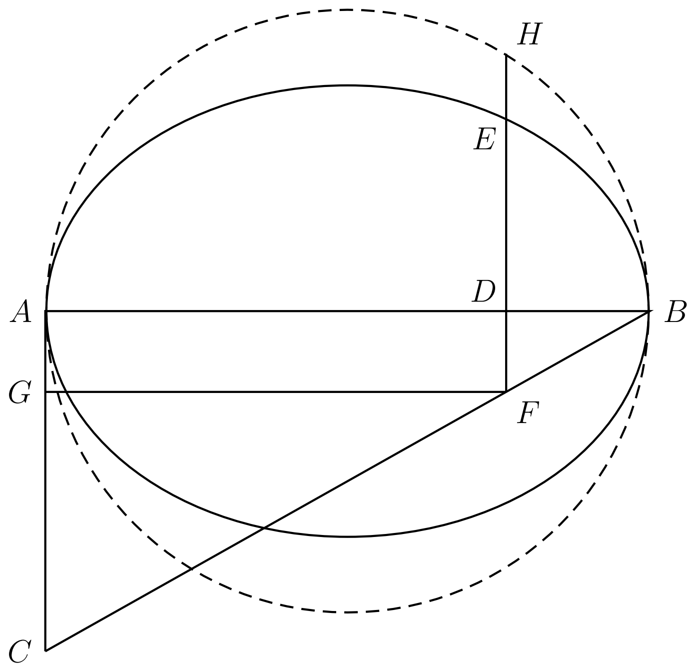
Problem 1. The ellipse AEB is determined as follows. Triangle ABC is given, the angle at A being right. If a point D is chosen at random on AB, and DE is erected at right angles to AB, then E lies on the ellipse if (and only if) the square on DE is equal to the rectangle $ADFG$ (which is formed by letting ED, extended as necessary, meet BC at F). Let also the circle AHB with diameter AB be given.

Find h (in terms of the given straight lines) such that h is to AB as the ellipse is to the circle. Prove that your answer is correct, using Newton's lemmas as needed.



Problem 2. We have used without proof Propositions I.33 and 49 of the *Conics* of Apollonius. This problem is an opportunity to prove those propositions, using the techniques of Descartes and Newton as appropriate.

A straight line ℓ (not shown), a curved line ABE, and a straight line AC are given such that, whenever a point B is chosen at random on ABE, and straight line BC is dropped perpendicular to AC, then the square on BC is equal to the rectangle bounded by ℓ and AC. So ABE is a parabola with axis AC.

Let B now be fixed; so we may write $BC = a$ and $AC = b$. Extend CA to D so that $AD = AC$. Draw straight line DBK, and let $c = BD$.

Let a point E be chosen at random on the parabola ABE. Draw straight lines BF parallel to AC, and EF parallel to BD.

(a). Show that the parabola ABE must indeed lie all on one side of DBK.

(b). Show that the square on EF varies as BF, and find m (in terms of a, b, and c only) such that $m \times BF$ is equal to the square on EF. For your computations, let $x = EF$ and $y = BF$.

(c). Explain why BD is tangent to the parabola at B.