Antalya Algebra Days XIV
in Çeşme

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Contents

Invited talks

How many rational points can a high genus curve over a finite field have?

Alp Bassa

In this talk we will be interested in the question of how many rational points a high genus curve over a finite field can have. We will introduce several approaches to this problem and present a recent result (joint work with Beelen, Garcia, Stichtenoth) over all non-prime finite fields.

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The Hrushovski Programme

Alexandre Borovik

The aim of the talk is to discuss an approach to classification of simple groups of finite Morley rank via study of groups of fixed points of their generic automorphisms. It has been proposed by Udi Hrushovski (about a decade ago) and promises a synthesis of the theories of finite groups and algebraic groups with the model theory—as well as new insights into the nature of classification of finite simple groups—much deeper than the ones currently achieved in the theory of groups of finite Morley rank.

I will explain some recent results by Omaima Alshanqiti, Pınar Uğurlu, and Şükrü Yalçınkaya closely related to this programme.

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A functor approach to modular representations of GL_n

Marcin Chałupnik

I will explain how natural constructions in linear algebra help to understand representations of the general linear groups. We will focus on homological problems such us computing Ext-groups between modular representations of GL_n. I will describe combinatorial structures governing these Ext-groups and discuss a surprising connection between modular representations of GL_n and representations of Kac-Moody algebras of type A_n.

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Exponential polynomials

Paola D'Aquino

I will consider exponential polynomials over algebraically closed fields of characteristic 0 with an exponentiation. I will describe a factorization theorem for such polynomials extending a result of Ritt [2]. I will also examine some consequences of Schanuel's Conjecture in transcendental number theory for exponential polynomials over the complex field, and more in general over the exponential fields introduced by Zilber [3]. In particular, I will relate Schanuel's Conjecture to another conjecture due to Shapiro going back to 1956 on a system of two exponential polynomials.

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Pairs of algebraically closed fields

Françoise Delon

A "pair of algebraically closed field" consists of an algebraically closed field enriched with an additional unary predicate interpreted as an algebraically closed subfield. We propose a language in which they eliminate quantifiers, and which has the advantage of adapting to some expansions. We consider more precisely dense and "separated" pairs of algebraically closed valued fields. The latter had been axiomatized by Baur, who had also proved that any pair in which the small field is maximal is separated.

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Polynomial-Exponential Equations

Ayhan Günaydın

We consider polynomial-exponential equations over complex numbers where the variables run through rational numbers. Classically, the integer solutions of such equations are considered and there are several finiteness results in the literature for those solutions (for instance [2] and [3]). We present a method to reduce the rational solutions to integer ones and give a description of them using the earlier results. As a corollary, we get a finiteness result. If time permits, we present connections with the Mordell-Lang Conjecture.

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Valued difference fields and the tree property of the second kind

Martin Hils

In model theory, important dividing lines are usually defined in therms of combinatorial properties of definable sets, e.g. stable and NIP theories are defined in this manner. Recall that a formula $\phi(x, y)$ is said to have the independence property, if one may shatter arbitrarily large finite sets, using instances $\phi(x, a)$ of ϕ. A theory is *NIP* if no formula has the independence property. Valued fields provide examples par excellence of unstable NIP theories: (the theory of) every algebraically closed valued fields is NIP, as is any henselian valued field of residue characteristic 0, provided the residue field is NIP.

Combining Hrushovski's very deep results on the non-standard Frobenius automorphism [3] with the work of Azgın on valued fields with a contracting automorphism [1], one may obtain an axiomatisation of the first-order structure given by an algebraically closed valued field of residue characteristic 0 equipped with a non-standard Frobenius automorphism. This structure, a *valued difference field*, is not NIP, since the induced field automorphism on the residue field is 'generic'. But one may show that it is next best: it does not have the *tree property of the second kind*, i.e. is NTP_2. More generally, in the context of an Ax-Kochen-Ersov principle for valued difference fields (see [1]), NTP_2 transfers from the value group (with automorphism) and the residue difference field to the valued difference field itself.

The property NTP_2 had already been introduced by Shelah in 1980, but only recently it has been shown to provide a fruitful 'tameness' assumption, e.g. when dealing with independence notions in unstable NIP theories (work of Chernikov and Kaplan [2]).

In the talk, all the above notions will be defined and put into a larger context. Moreover, we will sketch the proof of our main result, namely that certain valued difference fields are NTP_2. This is joint work with Artem Chernikov.

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Multiple zeta values from L. Euler to F. Brown

Amir Jafari

In this expository talk I will explain the relation between multiple zeta values, defined by Euler centuries ago, and recent topics such as Hodge, p-adic and motivic periods. I will also explain the relations that such numbers satisfy and the relation between them and the motivic fundamental group of the sphere minus three points.

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Automorphism Groups of Rational Elliptic Surfaces with Section

Tolga Karayayla

The aim of this talk is to describe the classification of the automorphism groups of relatively minimal rational elliptic surfaces with section defined over the base field $\mathbb{C}$. Let B be such an elliptic surface and $Aut(B)$ be the group of automorphisms of B (biholomorphic maps on the complex manifold B). As an elliptic surface, B has a projection map $\beta : B \to \mathbb{P}^1$ to the complex projective line $\mathbb{P}^1$ such that the generic fiber is an elliptic curve and there are finitely many singular fibers which can be of certain types. An equivalent description of B is that it can be obtained from the projective plane $\mathbb{P}^2$ by blowing up the 9 base points of a pencil of generically smooth cubics. The configuration of the singular fibers gives important information about $Aut(B)$. Oguiso and Shioda [2] have shown that the Mordell-Weil group $MW(B)$ of B (the group of the sections of the surface), which naturally embeds in $Aut(B)$, is determined by the configuration of the singular fibers on B. In [1], we show that $Aut(B) = MW(B) \rtimes Aut_\sigma(B)$ where $Aut_\sigma(B)$ denotes the subgroup of the automorphisms of B preserving the zero section σ, and for the surfaces B with non-constant J maps we list all possible groups which can arise as $Aut_\sigma(B)$ corresponding to each configuration of singular fibers on B. In this presentation I will show how the configuration of singular fibers on the surface gives some criteria on $Aut_\sigma(B$ and how these criteria can be used to determine the possible groups $Aut_\sigma(B)$.

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On model-theoretic connected components in some group extensions

Krzysztof Krupiński

This is joint work with Jakub Gismatullin.

For a group G definable in a monster model and for a small set of parameters B, we denote by G_B^{00} the smallest B-type-definable subgroup of bounded index and by G_B^{000} the smallest B-invariant subgroup of bounded index. It was an open problem to find a group G for which $G_B^{00} \neq G_B^{000}$. The first example, found by Conversano and Pillay, is the universal cover $\widetilde{\mathrm{SL}_2(\mathbb{R})}$ of $\mathrm{SL}_2(\mathbb{R})$. Their proof uses the fact that $\widetilde{\mathrm{SL}_2(\mathbb{R})}$ is a central extension of $\mathrm{SL}_2(\mathbb{R})$ by $\mathbb{Z}$ given by a definable 2-cocycle $h\colon \mathrm{SL}_2(\mathbb{R}) \times \mathrm{SL}_2(\mathbb{R}) \to \mathbb{Z}$ with finite image, and also the facts that both groups $\mathrm{SL}_2(\mathbb{R})$ and $\widetilde{\mathrm{SL}_2(\mathbb{R})}$ are perfect (i.e. equal to their commutator subgroups).

This led us to the following general question.

Question. *When does an extension $\widetilde{G}$ of a group G by an abelian group A satisfy $\widetilde{G}_B^{00} \neq \widetilde{G}_B^{000}$ for some parameter set B (working in a monster model)?*

We consider this problem in a general algebraic context, i.e. without assuming that $\widetilde{G}$ is a universal cover of a topological group or that G is definable in an o-minimal structure. The only restriction that we make is the assumption that the 2-cocycle $h\colon G \times G \to A$ defining our extension is definable and has finite image.

Our goal was to find sufficient (and necessary, at least in some situations) conditions on h for which $\widetilde{G}_B^{00} \neq \widetilde{G}_B^{000}$, and our main theorem provides such conditions.

Using this theorem, we obtain new classes of examples of extensions (including the example of Conversano and Pillay) for which $\widetilde{G}_B^{00} \neq \widetilde{G}_B^{000}$, e.g. some central extensions of $\mathrm{SL}_2(k)$ for k being any ordered field. In order to apply our theorem to get these new examples, we use Matsumoto-Moore theory.

During my lecture, I will discuss the main theorem, and, if time permits, I will present some of the examples which we have obtained applying our theorem.

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Automatic quantifier elimination and mutually algebraic structures

Chris Laskowski

A series of results indicate that sufficiently strong model theoretic hypotheses imply a bound on the quantifier complexity, regardless of the presentation of the model. For example, the elementary diagram of any model of a trivial, strongly minimal theory is model complete. We are now able to understand such examples by introducing the notions of mutually algebraic formulas, theories, and structures. We prove that every structure has a mutually algebraic hull and give a number of characterizations of a theory being mutually algebraic. The most striking equivalence is that a theory T is mutually algebraic if and only if no expansion of a model of T by adding unary predicates has the finite cover property.

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Valued difference fields

Gönenç Onay

Let (K, v) be a valued field with a distinguished automorphism σ which preserves the valuation ring $\mathcal{O}_K$, hence inducing automorphisms: σ_v on the ordered value group of (K, v) and $\bar{\sigma}$ on the residue field of (K, v). In [1], S.Durhan (formerly S.Azgın) considered the case where σ_v is contractive (: $\sigma_v(\gamma) > n\gamma \quad \forall\gamma > 0$ and $\forall n \in \mathbb{N}$); in [3], K.Pal considered the case where σ_v is multiplicative (: $\sigma_v : \gamma \mapsto \rho\gamma$, for some $\rho > 0$ in some real closed field) while my thesis (cf. [2]) involves study of σ-linear equations (i.e. equations of the form $\sum_i a_i\sigma^i(x) = b$), where σ_v is auto-increasing (: $\sigma_v(\gamma) > \gamma$ for $\gamma > 0$).

In this talk, after recalling these results, I will present some new ones on the way of Ax-Kochen and Ershov type theorems which permit one to recognise the first order theory of the *valued difference field* (K, v, σ) by those of its value group and residue field **with no assumption on** σ_v while keeping already present hypotheses in above works on residue field. This is an ongoing work *joint with* Salih Durhan.

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Alternatives for pseudofinite groups.

Françoise Point

I will present a joint work ([1]) with A. Ould Houcine on alternatives for pseudofinite groups of the same flavour as the Tits alternative for linear groups.

We show that an $\aleph_0$-saturated pseudofinite group either contains the free subsemigroup of rank 2 or is nilpotent-by-(uniformly locally finite). We strengthen the result to pseudofinite groups satisfying a finite disjunction of Milnor identities by showing that such group is nilpotent-by-(uniformly locally finite). Then we show that whether the following dichotomy holds for $\aleph_0$-saturated pseudo-finite groups, namely it either contains a free non abelian subgroup or it is amenable, is equivalent to whether a finitely generated residually finite group which satisfies a non-trivial identity is amenable (respectively uniformly amenable).

A class of finite groups is *weakly of bounded r-rank* if the class of the radicals is of r-bounded (Prüfer) rank and the index of the sockels are r-bounded. We obtain the following dichotomies for an $\aleph_0$-saturated pseudo-(finite weakly of bounded rank) group G: either G contains a nonabelian free group or G is nilpotent-by-abelian-by-(uniformly locally finite). This strenghtens former results of S. Black [2] who considered a "finitary Tits alternative" (for a class of finite groups).

Using a result of E. Khukhro ([3]) on classes of finite soluble groups satisfying some uniform conditions on centralizer dimension, we show that an $\aleph_0$-saturated pseudo-(finite of bounded centralizer dimension) group either contains a nonabelian free group or is soluble-by-(uniformly locally finite).

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The diameter of permutation groups

Ákos Seress

For a group G and a set of generators S of G, the *Cayley graph* $\Gamma(G, S)$ is defined to have vertex set G and $g, h \in G$ are adjacent if and only if $gs = h$ or $hs = g$ for some $s \in S$. The *diameter* of $\Gamma(G, S)$ is the maximum distance among pairs of vertices; equivalently, the diameter is the minimum number d such that every group element can be written as a word of length at most d in terms of the elements of S and their inverses. The diameter problem may be interesting for a particular group and set of generators (how many turns do we need to solve Rubik's cube?), but the mathematically most challenging questions are about estimating

$$diam(G) := max_S\{diam(\Gamma(G, S))\}$$

with the maximum taken over all sets of generators of G, and for G in an appropriate family of groups.

The challenge driving most recent activities is Babai's conjecture, which states that for all finite nonabelian simple groups, $diam(G) < (\log |G|)^c$, for some absolute constant c. The conjecture was proven by Pyber, Szabó and Breillard, Green, Tao in 2011 for Lie-type groups of bounded rank, but the case of alternating groups cannot be handled by their machinery. For alternating groups A_n, Babai's conjecture requires a polynomial, n^c, diameter bound. We can prove a slightly weaker quasipolynomial result:

$$diam(A_n) < \exp(O((\log n)^4 \log\log n)).$$

This is joint work with Harald Helfgott (ENS, Paris).

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The Bloch-Beilinson Conjectures

Vasudevan Srinivas

The Bloch-Beilinson Conjectures are some of the deepest open questions in mathematics today, relating aspects of algebraic geometry, algebraic K-theory and number theory.

The conjectures have roots, on the one hand, in classical results (Euler, Riemann, Dedekind, Hilbert, Artin, etc.) on special values and zeroes of zeta functions, in the period upto the early 20th century. Another source, somewhat more recent (going upto the mid 1970's) is work of Tate, Iwasawa, Lichtenbaum, Quillen and Borel, which brought in the role of algebraic K-theory.

The most recent inspiration, beginning with several key calculations of Bloch, relate these to algebraic geometry. Bloch's vision was articulated in a general, more precise form by Beilinson, around 1982, resulting in what we now call the Bloch-Beilinson Conjectures. There are also refinements (e.g. the Bloch-Kato conjectures).

In fact there is tantalising, but rather meagre, evidence to support these conjectures, in spite of some 30 years of effort by mathematicians. Some new insights seems to be needed, to lead to a solution of these open questions!

My lecture will give an introduction to this important circle of ideas. Some references are provided for further study.

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Decidable and undecidable real closed rings

Marcus Tressl

Real closed fields are ordered fields satisfying the intermediate value property for polynomials. Tarski showed that these fields are precisely those which have the same first order theory as the field of real numbers. It is a formal consequence of this result that the truth of first order sentences of the field of real numbers can be verified by a computer (this property is called *decidability* of the field of real numbers).

Real closed rings occur in the topological study of *semi-algebraic sets*, i.e. sets described by polynomial inequalities (e.g. the closed unit disc is such a set). Real closed rings play a similar role in the class of partially ordered rings as real closed fields play in the class of ordered fields. To name some examples of real closed rings: the ring of real valued continuous (semi-algebraic) functions defined on the unit disc is a real closed ring. Also the ring of germs of continuous functions about a point, and the ring of germs at ∞ of bounded functions on the real line is real closed (the latter is also a valuation ring).

I will give precise definitions and say what happens with Tarski's decidability result in various real closed rings. Here are some (local) examples:

- Convex subrings of real closed fields are real closed and decidable by Cherlin-Dickmann.
- Rings of germs of continuous functions about a point are decidable if and only if the ambient space is of dimension 1.
- This example uses some terminology from model theory, to be explained in the talk: Every *pair* of real closed fields is bi-interpretable with a real closed ring; we know by Baur and Macintyre that there are undecidable such pairs, but we also have decidable pairs such as dense pairs (e.g. real algebraic numbers sitting in the reals) or tame pairs (e.g. the reals sitting in a non-standard real closed field).

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Contributed Talks

Generalized Skew Derivations on Lie Ideals of Prime Rings

Emine Albaş

This is a joint work with N. Argaç (Ege University), V. De Filippis (University of Messina), Ç. Demir (Ege University).

In this talk, R will represent an associative ring with center $Z(R)$, Q its right Martindale quotient ring, and C its extended centroid. Let α be an automorphism of R. An additive mapping $D : R \to R$ is called an *α-derivation* (or a *skew derivation*) on R if

$$D(xy) = D(x)y + \alpha(x)D(y)$$

for all $x, y \in R$. In this case, α is called the *associated automorphism* of D. Let $b \in Q$ be a fixed element. Then it is easy to see that the mapping $D : R \to R$ defined by $D(x) = bx - \alpha(x)b$, $x \in R$, is an α-derivation. Such an α-derivation is called an *inner α-derivation* (an *inner skew derivation*) defined by b. If a skew derivation D is not inner, then it is *outer*.

An additive mapping $F : R \to R$ is called a *generalized skew derivation* on R if there exists a skew derivation D of R with associated automorphism α such that

$$F(xy) = F(x)y + \alpha(x)D(y)$$

for all $x, y \in R$.

Many researchers investigated generalized skew derivations satisfying certain algebraic conditions from various point of views ([1], [3]). More recently in [3], Chou and Liu continued the line of investigation concerning the Engel-conditions $[F(x), x]_k = 0$ for all $x \in S$, a suitable subset of R, with F additive mapping in R. More precisely, if $S = L$ denotes a non-central Lie ideal of R, they proved the following: If R is a prime ring, d a non-zero skew derivation of R, and $k \geqslant 1$ a fixed integer such that $[d(x), x]_k = 0$, for all $x \in L$, then $char(R) = 2$ and R satisfies s_4, the standard identity in 4 variables.

By continuing the same line of investigation, we obtained the following:

Theorem. *Let R be a prime ring, Q its two-sided Martindale quotient ring, C its extended centroid, L a non-central Lie ideal of R, $F : R \to R$ be a nonzero generalized skew derivation of R and $k \geqslant 1$ a fixed integer. If $[F(u), u]_k = 0$, for all $u \in L$ then either there exists $\lambda \in C$ such that $F(x) = \lambda x$, for all $x \in R$, or R satisfies s_4 and one of the following holds:*

1. *$char(R) = 2$;*
2. *there exist $a \in Q$ and $\lambda \in C$ such that $F(x) = ax + xa + \lambda x$, for all $x \in R$.*

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Curves of Every Genus with a Prescribed Number of Rational Points

Nurdagül Anbar

This is a joint work with Henning Stichtenoth. A fundamental problem in the theory of curves over finite fields is to determine the sets

$$\mathcal{M}_q(g) := \{N \in \mathbb{N} \mid \text{ there is a curve over } \mathbb{F}_q \text{ of genus } g \text{ with exactly } N \text{ rational points.}\}$$

A complete description of $\mathcal{M}_q(g)$ is out of reach. So far, mostly *bounds* for the numbers $N_q(g) := \max \mathcal{M}_q(g)$ have been studied. In particular, Elkies et al. proved that there is a constant $\gamma_q > 0$ such that for any $g \geqslant 0$ there is *some* $N \in \mathcal{M}_q(g)$ with $N \geqslant \gamma_q g$. This implies that $\liminf_{g\to\infty} N_q(g)/g > 0$, and solves a long-standing problem by Serre. We extend the result of Elkies et al. substantially and show that there are constants $\alpha_q, \beta_q > 0$ such that for all $g \geqslant 0$, the *whole interval* $[0, \alpha_q g - \beta_q] \cap \mathbb{N}$ is contained in $\mathcal{M}_q(g)$.

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On Fitting p-groups with all proper subgroups satisfying an outer commutator law

Ahmet Arıkan

This is a joint work with Aynur Arıkan. In this talk we consider certain Fitting p-groups in which every proper subgroup satisfies an outer commutator identity and obtained some conditions for such groups to be imperfect. We also give an application of the main theorem to obtain an idea of the abundance of the groups under consideration.

The following key result due to Khukhro and Makarenko will play a crucial role to obtain the main results in the talk.

Theorem 1. ([1, Theorem 1],[2, Theorem 1] or [3]) *If a group G has a subgroup H of finite index n satisfying the identitiy*

$$\omega(\underbrace{H,\dots,H}_{w}) = 1,$$

where ω is an outer commutator word of weight w, then G has also a characteristic subgroup C of finite (n,w)-bounded index satisfying the same identity

$$\omega(\underbrace{C,\dots,C}_{w}) = 1.$$

We say that a group G has the property $\mathcal{EI}$ if for every finitely generated proper subgroup W of G and for every element a in $G \setminus W$, there is a finitely generated subgroup V containing W, a generating subset Y and a proper subgroup L of G such that

$$a \in \Big(\bigcap_{y \in Y \setminus L} \langle V, y\rangle \Big) \setminus V.$$

We call Y an associated set for $\mathcal{EI}$ with respect to W. Clearly in this case $a \notin V$, but $a \in \langle V, y\rangle$ for all $y \in Y \setminus L$.

Here are the main results which will be introduced in the talk.

Theorem 2. *Let G be a countable Fitting p-group with the property $\mathcal{EI}$ such that G is the associated set with respect to every finite subgroup. If for every proper subgroup K of G, there exits an outer commutator word ω of weight $\geqslant 2$ such that $K \in \mathfrak{X}_\omega$, then $G' \neq G$.*

Theorem 3. *Let G be a non-trivial locally nilpotent p-group with all proper normal subgroups soluble. Assume that for every proper subgroup K of G there exits an outer commutator word ω of weight $\geqslant 2$ such that $K \in \mathfrak{X}_\omega$. If G contains a proper subgroup U such that $|N : N \cap U|$ is finite for every proper normal subgroup of G, then*

$$G \neq G'.$$

Furthermore if $U \in \mathfrak{X}_u$ for some outer commutator word u of weight $\geqslant 2$, then $\gamma_3(G) \in \mathfrak{X}_u$.

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Unique Decomposition for Reduced Commutative Noetherian Rings

Başak Ay

A class C of modules is said to have the Krull-Schmidt property if every module in C is a direct sum of indecomposable members of C, and such a direct decomposition is unique up to isomorphism and order of the indecomposable summands. Let R be a reduced commutative Noetherian ring. In [1], the authors characterize R satisfying the Krull-Schmidt property for ideals. In the first part of this talk, this characterization will be provided together with some examples. In the last part of this talk, we show that if R is both local and one-dimensional satisfying the Krull-Schmidt property for ideals, then it has the Krull-Schmidt property for direct sums of rank one modules. We end the talk with the conjecture that the latter should also hold even R is not necessarily local.

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Model theory and nilpotence in groups with bounded chains of centralizers

Paul Baginski

A group G has bounded chains of centralizers ($\mathfrak{M}_C$) if every chain of centralizers of arbitrary subsets of G stabilizes after finitely many steps. Many classic groups from group theory possess the $\mathfrak{M}_C$ property, as do stable groups from model theory. However, unlike stability, $\mathfrak{M}_C$ is not an elementary property, nor is it preserved under quotients, even particularly natural ones. This frustrates many classical lines of proof from group theory and logic. We will discuss recent advances showing that despite these logical obstacles, the class of $\mathfrak{M}_C$ groups possess many definable subgroups. In particular, we shall demonstrate the construction of a descending chain of definable subgroups above any subgroup H of G; when H is nilpotent, this construction allows us to find a definable nilpotent envelope of H.

This talk reports on continuing work between the speaker and Tuna Altinel.

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Generalized α-derivations on Lie ideals with annihilating conditions

Nihan Baydar Yarbil

The talk will focus on a recent progress in a joint work with N.Argaç.

An additive map d from R into itself satisfying the rule $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$, is called *a derivation of* R. Let α be an automorphism of R. An $\alpha-$derivation of R is an additive mapping d satisfying $d(xy) = d(x)y + \alpha(x)d(y)$ for all $x, y \in R$. $\alpha-$derivations are sometimes called *skew derivations*. An additive mapping $f : R \to R$ is called generalized $\alpha-$derivation if there exists a $\alpha-$derivation $d : R \to R$ such that $f(xy) = f(x)y + \alpha(x)d(y)$ for all $x, y \in R$. Let α and β be automorphism of R then by a generalized (α, β)-derivation; we mean an additive map from R into itself such that $f(xy) = f(x)\alpha(y) + \beta(x)d(y)$ for all $x, y \in R$, where d is an (α, β)-derivation.

I.N. Herstein in [5] proved that if d is a derivation of a prime ring R such that $d(x)^n = 0$ for all $x \in R$, where $n \geqslant 1$ is a fixed integer, then $d = 0$ and the annihilating condition for the same case is studied by M.Brešar in [1]. He proved that if $ad(x)^n = 0$ for all $x \in R$, where $n \geqslant 1$ is a fixed integer,then $a = 0$ provided $charR = (n-1)!$.

Later in [6] Lee and Lin obtained the same conclusion assuming that $ad(x)^n = 0$ for all x in some noncentral Lie ideal of R without the assumption on characteristic.

In [4], C. M. Chang and T. K. Lee proved the following: Let R be a prime ring, L a noncommutative Lie ideal of R and d a nonzero derivation of R and $0 \neq a \in R$. Suppose that $ad(x)^n \in Z(R)$ for all $x \in L$, where n is a fixed positive integer. Then $dim_C RC = 4$.

When it comes to generalized α- derivations in prime rings, the case $f(x)^n \in Z(R)$ for all $x \in I$, a nonzero ideal of R, and for some fixed positive integer n, is studied by I.C. Chang in [3]. In this case he proved that R is either commutative or is an order in a 4-dimensional simple algebra.

More recently, in [2], J.C. Chang proved the following: Let R be a prime ring, f a generalized α- derivation and $a \in R$. If $af(x)^n = 0$ for all $x \in R$, where n is a fixed positive integer, then $af(x) = 0$ for all $x \in R$. Moreover, if $d \neq 0$ or $f \neq 0$, then $a = 0$.

Motivated by these results, our main objective in this talk is to describe the situation when the problem above is studied for Lie ideals. More precisely, we have the following;

Theorem. Let R be a prime ring and α an automorphism of R. Let f be a generalized α-derivation of R and $a \in R$. Suppose that L is a noncommutative Lie ideal of R. If $af(x)^n = 0$ for all $x \in L$, where n is a fixed positive integer, then $af(x) = 0$ for all $x \in R$.

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Some rigid moieties of various relational homogeneous structures

Doğan Bilge

Given a countable set X, a *moiety* of X is a subset which is countable and co-countable. A *rigid* embedding of a structure M into a structure N is an embedding where each automorphism of M extends uniquely to an automorphism of N. We show the existence of rigid moieties in various homogeneous relational structures including universal K_n-free graphs, Henson's continuous family of digraphs and the universal structure in a finite relational language. We finally prove the following:

Theorem 1. *Let $\mathcal{K}$ be a not totally disconnected free amalgamation class in a finite relational language $\mathcal{L}$ and assume that all the one-point sets in $\mathcal{K}$ are isomorphic. Then every countably infinite $\mathcal{L}$-structure K, whose age lies in $\mathcal{K}$, can be embedded as a rigid moiety into the Fraïssé limit of $\mathcal{K}$, denoted $\mathbf{K}$. Moreover, there are 2^ω many such embeddings which are not conjugate in $Aut(\mathbf{K})$.*

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Derivatives of Bivariate Fibonacci Polynomials

Tuba Çakmak

This is a joint study with Erdal Karaduman.

In this study, it is given new algebraic properties related to bivariate Fibonacci polynomials. Also, we present the partial derivatives of this polynomials in the form of convolution of bivariate Fibonacci polynomials and we give asymptotic behaviour of the quotient of consecutive terms.

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On Products of Conjugacy Classes and Irreducible Characters in Finite Groups

M. R. Darafsheh

Let G be a finite group. For irreducible complex characters χ and φ of G the irreducible constituents of $\chi\varphi$ is denoted by $\eta(\chi\varphi)$. If A and B are two conjugacy classes in G, then AB is a union of conjugacy classes in G and $\eta(AB)$ denotes the number of distinct conjugacy classes of G contained in AB. In this paper we investigate the current research on the impact of these η-functions on the structure of G as well as some similarity between them.

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Superderivations of Associative Superalgebras

Çağrı Demir

Let $\mathbb{Z}_2$ denote the integers modulo 2. A $\mathbb{Z}_2$-graded associative algebra $\mathcal{A}$ over a unital commutative ring Φ is called an associative *superalgebra*. This means that there exist Φ-submodules $\mathcal{A}_0$ and $\mathcal{A}_1$ of $\mathcal{A}$ such that $\mathcal{A} = \mathcal{A}_0 \oplus \mathcal{A}_1$ and $\mathcal{A}_0\mathcal{A}_0 \subseteq \mathcal{A}_0$ ($\mathcal{A}_0$ is a subalgebra of $\mathcal{A}$), $\mathcal{A}_0\mathcal{A}_1 \subseteq \mathcal{A}_1$, $\mathcal{A}_1\mathcal{A}_0 \subseteq \mathcal{A}_1$ ($\mathcal{A}_1$ is an $\mathcal{A}_0$-bimodule), and $\mathcal{A}_1\mathcal{A}_1 \subseteq \mathcal{A}_0$. An element $a_i \in \mathcal{A}_i$, $i = 0$ or $i = 1$, is said to be homogeneous of degree i, and we write $|a_i| = i$ to indicate the homogeneity degree of a_i.

A *superderivation of degree* 0 is a Φ-linear map $d_0 : \mathcal{A} \to \mathcal{A}$ such that $d_0(\mathcal{A}_0) \subseteq \mathcal{A}_0$, $d_0(\mathcal{A}_1) \subseteq \mathcal{A}_1$ and $d_0(xy) = d_0(x)y + xd_0(y)$ for all $x, y \in \mathcal{A}_0 \cup \mathcal{A}_1$. A *superderivation of degree* 1 is a Φ-linear map $d_1 : \mathcal{A} \to \mathcal{A}$ such that $d_1(\mathcal{A}_0) \subseteq \mathcal{A}_1$, $d_1(\mathcal{A}_1) \subseteq \mathcal{A}_0$ and $d_1(xy) = d_1(x)y + (-1)^{|x|}xd_1(y)$ for all $x, y \in \mathcal{A}_0 \cup \mathcal{A}_1$. A *superderivation* $d : \mathcal{A} \to \mathcal{A}$ is a sum of a superderivation d_0 of degree 0 and a superderivation d_1 of degree 1.

The *supercommutator* of any given homogeneous elements $a, b \in \mathcal{A}_0 \cup \mathcal{A}_1$ is defined to be $[a, b]_s = ab - (-1)^{|a||b|}ba$. We then define the supercommutator of any pair of elements $a, b \in \mathcal{A}$ by linearly extending the above definition on homogeneous elements, that is $[a, b]_s = [a_0, b_0] + [a_0, b_1] + [a_1, b_0] + [a_1, b_1]_s$, where $a = a_0 + a_1$ and $b = b_0 + b_1$. Let $a = a_0 + a_1 \in \mathcal{A}$ be a fixed element and define the mapping $d : \mathcal{A} \to \mathcal{A}$ by $d(x) = ad_s(a)(x) = [a, x]_s$ for all $x \in \mathcal{A}$. Then d is a superderivation of $\mathcal{A}$ with $d_0(x) = [a_0, x]_s = [a_0, x]$ and $d_1(x) = [a_1, x]_s$ for all $x \in \mathcal{A}$. Such superderivations are called *inner superderivations*.

Let $\mathcal{A}$ be a unital associative superalgebra and $\mathcal{U}(\mathcal{A})$ denote the multiplicative group of units in $\mathcal{A}$. If d is a superderivation of $\mathcal{A}$ such that $d(\mathcal{A}) \subseteq \mathcal{U}(\mathcal{A}) \cup \{0\}$, then we say that d is a superderivation with zero or invertible values. In the present talk, we will mainly focus on the problem of describing the structure of unital superalgebras those having a nonzero superderivation with zero or invertible values. This problem is originally inspired by the works of Bergen, Herstein and Lanski in [1] and of Bergen and Herstein in [2] in which they initiated the study of certain kinds of mappings with zero or invertible values on arbitrary unital rings.

Our main result reads as follows:

Theorem. *Let $\mathcal{A} = \mathcal{A}_0 \oplus \mathcal{A}_1$ be a nontrivial unital superalgebra (i.e. $\mathcal{A}_1 \neq (0)$) over Φ and d be a nonzero superderivation of $\mathcal{A}$ such that $d(x)$ is either zero or invertible for all $x \in \mathcal{A}$. If Φ contains the element $\frac{1}{2}$, then $\mathcal{A}$ is either a division superalgebra $\mathcal{D}$, or $M_2(\mathcal{D})$, or it is a local superalgebra with a unique maximal graded ideal $\mathcal{M}$ such that $\mathcal{M}^2 = (0)$.*

We will also describe in details the local superalgebras that are possible.

This is a joint work with E. Albaş, N. Argaç and A. Fošner. The work has been supported by TÜBİTAK Grant #110T586.

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A Closure of Proper Classes Induced by a Class of Homomorphisms

Yılmaz Mehmet Demirci

Throughout, R is an associative ring with identity unless otherwise stated and modules are unital R-modules. Let $\mathfrak{F}$ and $\mathfrak{G}$ be families of homomorphisms of R-modules closed under compositions and $\mathcal{P}$ be a class of short exact sequences. We say that $(\mathfrak{F}, \mathfrak{G})$ a "*compatible*" pair for the class $\mathcal{P}$ if for every short exact sequence E, there is $f \in \mathfrak{F}$ such that $f^*(E) \in \mathcal{P}$ if and only if there is $g \in \mathfrak{G}$ such that $g_*(E) \in \mathcal{P}$ with one (or both) of the following conditions satisfied:

(i) $\mathfrak{F}$ is closed under pushout diagrams.

(ii) $\mathfrak{G}$ is closed under pullback diagrams.

For a class $\mathcal{P}$ of short exact sequences and a *compatible pair* $(\mathfrak{F}, \mathfrak{G})$ we define the class $\hat{\mathcal{P}}_{\mathfrak{F}}^{\mathfrak{G}}$ as

$$\begin{aligned}\hat{\mathcal{P}}_{\mathfrak{F}}^{\mathfrak{G}} &= \{E|f^*(E) \in \mathcal{P} \textit{ for some } f \in \mathfrak{F}\}\\ &= \{E|g_*(E) \in \mathcal{P} \textit{ for some } g \in \mathfrak{G}\}.\end{aligned}$$

Theorem 1. *For every proper class $\mathcal{P}$ and a compatible pair $(\mathfrak{F}, \mathfrak{G})$ for $\mathcal{P}$ the class $\hat{\mathcal{P}}_{\mathfrak{F}}^{\mathfrak{G}}$ is proper.*

In case $\mathfrak{F}$ and $\mathfrak{G}$ contain identity endomorphisms the class $\hat{\mathcal{P}}_{\mathfrak{F}}^{\mathfrak{G}}$ contains $\mathcal{P}$.

Over the ring $\mathbb{Z}$ of integers, for a proper class $\mathcal{P}$ the class $\hat{\mathcal{P}} = \{E \mid nE \in \mathcal{P}$ for some $0 \neq n \in \mathbb{Z}\}$ is a proper class is shown in [1]. For a class $\mathcal{R}$ and a positive integer k, we define the class $\hat{\mathcal{R}}_k$ as $\hat{\mathcal{R}}_k = \{E \mid k^t E \in \mathcal{R}$ for some positive integer $t\}$.

Proposition 2. *$\hat{\mathcal{P}}_k$ is a proper class for every proper class $\mathcal{P}$ and every positive integer k.*

Let us denote the quasi-splitting short exact sequences by $\mathcal{S}plit$. The direct sum of proper classes is defined in [2]. We have the following corollary using the same definition.

Corollary. $\hat{\mathcal{S}plit} = \bigoplus_p \hat{\mathcal{S}plit}_p$, *where p ranges over all prime numbers.*

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Valued Difference Fields and Valued Fields of Positive Characteristic

Salih Durhan

I will present the similarities surrounding valued difference fields and valued fields of positive characteristic. To answer model theoretic questions (which all the time boil down to algebraic questions), valued fields of positive characteristic have been analysed using global Galois theory. Such a strong tool is not available for valued difference fields and hence one is forced to use much more elementary methods. I will illustrate the basic tools and concepts which apply simultaneously to certain valued difference fields and valued fields of positive characteristic. These tools have been introduced in [1] but not yet utilized to their full power. I will mention some recent developments on the issue which bear the promise of an Ax-Kochen type result for the transseries field equipped with the right-shift automorphism (considered as a valued difference field). For positive characteristic valued fields same techniques can be applied, thus eliminating the non-constructive results stemming from Galois theory, to obtain a new understanding of tame fields whose model theoretic properties have been established by Franz-Viktor Kuhlmann.

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Homotopes of algebras

A.S. Dzhumadil'daev

Let $A = (A, \circ)$ be an algebra with vector space A and multiplication $\circ$. We consider operations on A derived by $\circ$ and some elements of A. For example, we endow A by a new multilication $\star$ given by $a \star b = (a \circ u) \circ b$ or $a \star b = (a \circ b) \circ u$ or $a \star b = (a \circ u) \circ (b \circ v)$, etc. Algebras consructed in a such way are called homotopes of algebra A. We show that these kind of operations may give us new interesting algebraic structures. We apply such approach for Novikov algebras, Leibniz algebras and Zinbiel algebras to construct new classes of non-associative algebras.

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SGQ-Projective Modules

Berke Kaleboğaz

Joined work with : Prof. Dr. Derya Keskin Tütüncü

In this work all rings are associative with identity and all modules are unitary right modules. S will denote the endomorphism ring of any module M, and $J(R)$ will denote the Jacobson radical of any ring R.

In this work we define the right ideal $D(s) = \{\varphi \in S \mid \mathrm{Im}\varphi \subseteq \mathrm{Im}s\}$ of S. Actually $D(s) = Hom(M,\mathrm{Im}s)$ for any endomorphism s of M. In this paper we introduce SGQ-projective modules by means of $D(s)$. Let M be any module. Then we call M SGQ-*projective* if for any $0 \neq s \in S$, there exists a right ideal X of S such that $D(s) = sS \oplus X$.

In this work, mainly, we prove the following facts:

Theorem 1: Let M be an SGQ-projective module. Then

(1)$\nabla \subseteq J(S)$

(2)If every proper submodule of M is contained in a proper co-M-cyclic submodule of M, then $\nabla = J(S)$.

Theorem 2: Let M be a weakly supplemented SGQ-projective π-projective module. Then S is regular if and only if $\nabla = 0$.

Theorem 3: The following are equivalent for a module M:

(1)Every R-module is SGQ-projective.

(2)Every R-module is semi-Hopfian.

(3)R is semisimple.

Theorem 4: Let M be any module and K any direct summand of M. If M is SGQ-projective, then K is SGQ-projective.

Any direct sum of two SGQ-projective module need not be SGQ-projective.

Theorem 5: Let A_i be SGQ-projective for each $i \in I$ such that $M = \oplus_{i \in I} A_i$. If every A_i is fully invariant in M, then M is SGQ-projective.

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A Generalization of Cofinitely Lifting Modules

Burcu Nişancı Türkmen

(This is joint work with Ali Pancar) Throughout the whole text, all rings are to be associative, identity and all modules are left unitary. Let R be a ring and let M be an R-module. The notation $N \leqslant M$ means that N is a submodule of M. A submodule K of M is called *cofinite* (in M) if the factor module $\frac{M}{K}$ is finitely generated. A submodule S of M is called *small (in M)*, denoted as $S << M$, if $M \neq S + L$ for every proper submodule L of M. By $Rad(M)$ we denote the intersection of all maximal submodules of M. For any ring R, an R-module M is called *(cofinitely) supplemented* if every (cofinite) submodule N of M has a *supplement*, that is a submodule K minimal with respect to $M = N + K$. Equivalently, $M = N + K$ and $N \cap K << K$ [9].

A module M is called *lifting* if for every submodule N of M there exists a direct summand K of M such that $K \leqslant N$ and $\frac{N}{K} << \frac{M}{K}$. Mohamed and Müller has generalized the concept of lifting modules to $\oplus$-supplemented modules. M is called *$\oplus$-supplemented* if every submodule N of M has a supplement that is a direct summand of M [7]. Then, Çalışıcı and Pancar have defined a module M *$\oplus$-cofinitely supplemented* if every cofinite submodule of M has a supplement that is a direct summand of M [7].

Wang and Wu call a module M *cofinitely lifting* if every cofinite submodule N of M there exists a direct summand K of M such that $K \leqslant N$

and $\frac{N}{K} << \frac{M}{K}$ [14]. It is shown in [14, Proposition 2.4] that every cofinitely lifting module is $\oplus$-cofinitely supplemented.

Let M be an R-module and let N and K be any submodules of M. If $M = N + K$ and $N \cap K \subseteq Rad(K)$, then K is called a *Rad-supplement* of N in M [8]. Since $Rad(K)$ is the sum of all small submodules of K, every supplement submodule is a Rad-supplement in M. One calls a module M *(cofinitely) Rad-supplemented* if every (cofinite) submodule has a Rad-supplement in M as in [2] and [5]. On the other hand, M is called *(cofinitely) Rad-$\oplus$-supplemented* if every (cofinite) submodule of M has a Rad-supplement that is a direct summand of M ([5] and [11]).

Recall from Al-Khazzi and Smith [1] that a module M is said to have *the property (P^*)* if for every submodule N of M there exists a direct summand K of M such that $K \leqslant N$ and $\frac{N}{K} \subseteq Rad(\frac{M}{K})$. Radical modules have the property (P^*). It is clear that every lifting module has the property (P^*) and every module with the property (P^*) is Rad-$\oplus$-supplemented.

As motivated by the above definitions, it is natural to introduce a generalization of modules with the property (P^*). We say that a module M is *cofinitely Rad-lifting* if for every cofinite submodule N of M there exists a direct summand K of M such that $K \leqslant N$ and $\frac{N}{K} \subseteq Rad(\frac{M}{K})$. A module with the property (P^*) is cofinitely Rad-lifting. Also, a finitely generated cofinitely Rad-lifting is lifting. It is clear that every cofinitely lifting module is cofinitely Rad-lifting.

In this study, we provide the properties of cofinitely Rad-lifting modules. Some examples are given to separate cofinitely lifting modules, cofinitely Rad-lifting modules and modules with the property (P^*). We show that a cofinitely Rad-lifting module which has a small radical is cofinitely lifting. We give some conditions for direct summands of a cofinitely Rad-lifting to be cofinitely Rad-lifting. We prove that a π-projective cofinitely (Rad-) $\oplus$-supplemented module is cofinitely (Rad-) lifting. We obtain a new characterization of semiperfect rings by using this result.

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Normal Subgroups in Models of Arithmetic

Ermek Nurkhaidarov

We will be presenting our joint with Erez Shochat research on closed normal subgroups of the automorphism group of saturated model of arithmetic. Let M be a saturated model of Peano Arithmetic of cardinality λ. We can consider the automorphism group of $M(Aut(M))$ as a topological group by letting the stabilizers of subsets of M of cardinality less than λ be the basic open subgroups.

Let I be a cut in a model M. We say I is *invariant* if for every $f \in Aut(M)$, $f(I) = I$. It is not difficult to see that if I is an invariant cut, then $Aut(M)_{(I)}$ (the pointwise stabilizers of I) is a closed normal subgroup in automorphism group of M. Kaye [1] shows that in countable recursively saturated models the converse is true. [2] proves that result for saturated models in weaker topology. We prove the similar result for saturated models of PA:

Theorem. *Let $H \leqslant Aut(M)$. Then H is a closed normal subgroup in $Aut(M)$ iff there exists an invariant cut $I \subset M$ such that $H = Aut(M)_{(I)}$.*

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Base Change

Gülümsen Onarlı

Within the theory of modules and more generally of Abelian categories, there is a very important set of results known as Morita Theory describing between categories of modules. The idea is that let $\phi : S \to R$ be a ring homomorphism and let M be a R-module, then we can obtain S-module $\phi^* (M)$ by means of ϕ for which the action is given by $s \cdot m = \phi(s)m$, for $s \in S, m \in M$. Then there is a functor

$$\phi^* : \mathbf{Mod}/R \longrightarrow \mathbf{Mod}/S\,.$$

This functor has a left adjoint

$$\phi_* : \mathbf{Mod}/S \longrightarrow \mathbf{Mod}/R\,.$$

Then each S-module N defines a R-module $\phi_*(N) = R \otimes_S N$. This construction is also known as "change of base" in a module theory. In this section we will see the corresponding idea with 2-crossed modules.
This is joint work with Ummahan Ege Arslan.

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Galois Cohomology, Spectral Sequences, and Local Class Field Theory

Matteo Paganin

In this talk, I will introduce an extension of the well known Lindon-Hochschild-Serre Spectral Sequence for a profinite group G and an open normal subgroup H that also takes into account the Tate Cohomology of the finite group G/H. Under certain conditions, this spectral sequence converges to zero. In particular situations, this fact implies the existence of families of isomorphisms and long exact sequences. In the case of a finite Galois extension of local fields, with Prof. David Vauclair of the University of Caen Basse-Normandie, we proved that the results obtained provide a different interpretation of the Reciprocity Map of the Local Class Field Theory.

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Fibres of coverings of curves over finite fields

Özgür Deniz Polat

We consider finite separable coverings of curves $f : X \to Y$ over a field of characteristic p. We are interested in describing the fibres of this cover in terms of double BwH cosets for some subgroups B, H of the monodromy group of f. We also compute the cardinality of fibres of a point for certain types of covering.

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Simple Polyadic Groups

M. Shahryari

Collaborated with: H. Khodabandeh

Let G be a non-empty set and n be a positive integer. If $f : G^n \to G$ is an n-ary operation, then we use the compact notation $f(x_1^n)$ for the elements $f(x_1, \ldots, x_n)$. In general, if $x_i, x_{i+1}, \ldots, x_j$ is an arbitrary sequence of elements in G, then we denote it as x_i^j. In the special case, when all terms of this sequence are equal to a constant x, we show it by $\overset{(t)}{x}$, where t is the number of terms. During this note, we assume that $n \geqslant 2$. We say that an n-ary operation is *associative*, if for any $1 \leqslant i < j \leqslant n$, the equality

$$f(x_1^{i-1}, f(x_i^{n+i-1}), x_{n+i}^{2n-1}) = f(x_1^{j-1}, f(x_j^{n+j-1}), x_{n+j}^{2n-1})$$

holds for all $x_1, \ldots, x_{2n-1} \in G$. An n-ary system (G, f) is called an *n-ary group* or a *polyadic group*, if f is associative and for all $a_1, \ldots, a_n, b \in G$ and $1 \leqslant i \leqslant n$, there exists a unique element $x \in G$ such that

$$f(a_1^{i-1}, x, a_{i+1}^n) = b.$$

One of the most fundamental theorems of polyadic group is the following, now known as *Hosszú -Gloskin's theorem*. We will use it frequently in this article and the reader can use [1] for detailed discussions.

Theorem 1. *Let (G, f) be an n-ary group. Then*

1. *on G one can define an operation $\cdot$ such that $(G, \cdot)$ is a group,*
2. *there exist an automorphism θ of $(G, \cdot)$ and $b \in G$, such that $\theta(b) = b$,*
3. *$\theta^{n-1}(x) = bxb^{-1}$, for every $x \in G$,*
4. *$f(x_1^n) = x_1\theta(x_2)\theta^2(x_3)\cdots\theta^{n-1}(x_n)b$, for all $x_1, \ldots, x_n \in G$.*

According to this theorem, we use the notation $der_{\theta,b}(G, \cdot)$ for (G, f) and we say that (G, f) is (θ, b)-derived from the group $(G, \cdot)$.

Before going to explanation of the motivations for the recent work, we recall the definition of normal polyadic subgroups from [1]. An n-ary subgroup H of a polyadic group (G, f) is called *normal* if

$$f(\overline{x}, \overset{(n-3)}{x}, h, x) \in H$$

for all $h \in H$ and $x \in G$. If every normal subgroup of (G, f) is singleton or equal to G, then we say that (G, f) is *group theoretically simple* or it is GTS for short. If $H = G$ is the only normal subgroup of (G, f), then we say it is *strongly simple in the group theoretic sense* or GTS^* for short. For any normal subgroup H of an n-ary group (G, f), we define the relation $\sim_H$ on G, by

$$x \sim_H y \iff \exists h_1, \dots, h_{n-1} \in H : \ y = f(x, h_1^{n-1}).$$

Now, it is easy to see that such defined relation is an equivalence on G. The equivalence class of G, containing x is denoted by xH and is called the *left coset* of H with the representative x. On the set $G/H = \{xH : x \in G\}$, we introduce the operation

$$f_H(x_1H, x_2H, \dots, x_nH) = f(x_1^n)H.$$

Then $(G/H, f_H)$ is an n-ary group derived from the group $ret_H(G/H, f_H)$, see [1]. One of the main aims of this article is to classify all GTS polyadic groups. We will give a necessary and sufficient condition for a polyadic group (G, f) to be GTS in terms of the ordinary group $(G, \cdot)$ and the automorphism θ. It is possible to define another kind of simpleness for polyadic groups, universal algebraically simpleness. Note that an equivalence relation R over G is said to be a *congruence*, if

1. $\forall i : \ x_i R y_i \Rightarrow f(x_1^n) R f(y_1^n)$,
2. $xRy \Rightarrow \overline{x} R \overline{y}$.

For example, if H is a normal polyadic subgroup of (G, f), then $R = \sim_H$ is a congruence, see [1]. We say that (G, f) is *universal algebraically simple* or UAS for short, if the only congruence is the *equality* and $G \times G$.

We prove the following two theorems concerning simple polyadic groups, see [3] for proofs.

Theorem 2. *(G, f) is UAS iff the only normal θ-invariant subgroups of $(G, \cdot)$ are trivial subgroups.*

Theorem 3. *A polyadic group (G, f) is GTS^* iff whenever K is a θ-invariant normal subgroup of $(G, \cdot)$ with θ_K inner, then $K = G$.*

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Products of homogeneous subspaces in free Lie algebras

Ralph Stöhr

Let L be a free Lie algebra of finite rank over a field K and let L_n denote the degree n homogeneous component of L. Formulae for the dimension of the subspaces $[L_m, L_n]$ for all m and n were obtained in [1]. In this talk I will report about recent work on subspaces of the form $[L_n, L_m, L_k] = [[L_m, L_n], L_k]$. Surprisingly, in contrast to the case of a product of two homogeneous components, the dimension of such products may depend on the characteristic of the field K. For example, the dimension of $[L_2, L_2, L_1]$ over fields of characteristic 2 is different from the dimension over fields of characteristic other than 2. Our main result are formulae for the dimension of $[L_m, L_n, L_k]$. Under certain conditions on m, n and k they lead to explicit formulae that do not depend on the characteristic of K, and express the dimension of $[L_m, L_n, L_k]$ in terms of Witt's dimension function. This is joint work with Nil Mansuroğlu.

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The Fibonacci Sequence in Finite Rings of Order p^2

Yasemin Taşyurdu

This is a joint work with İnci GÜLTEKİN.

In this paper, we obtain the period of Fibonacci sequence in finite 4 rings, up to isomorphism, having the presentations

$A = \langle a\ ;\ p^2 a = 0\ ,\ a^2 = a \rangle$,

$E = \langle a\ ;\ pa = pb = 0\ ,\ a^2 = a, b^2 = b,\ ab = a,\ ba = b \rangle$,

$F = \langle a\ ;\ pa = pb = 0\ ,\ a^2 = a, b^2 = b,\ ab = b,\ ba = a \rangle$,

$G = \langle a\ ;\ pa = pb = 0\ ,\ a^2 = 0, b^2 = b,\ ab = a,\ ba = a \rangle$

of order p^2 by using equality recursively defined by $F_{n+2} = A_1 F_{n+1} + A_0 F_n$, for $n \geqslant 0$, where $F_0 = 0$, $F_1 = 1$ and A_0 , A_1 are generator elements of these finite rings of order p^2. Also, we get some results between the period of the Fibonacci sequence in finite rings of order p^2 and characteristic of these rings.

The notion of Wall number was first proposed by D. D. Wall [1] in 1960 and gave some theorems and properties concerning Wall number of the Fibonacci sequence. K. Lü and J. Wang [8] contributed to the study of the Wall number for the -step Fibonacci sequence. D. J. DeCarli [6] gave a generalized Fibonacci sequence over an arbitrary ring in 1970. Special cases of Fibonacci sequence over an arbitrary ring have been considered by R. G. Buschman [4] , A. F. Horadam [2] and N. N. Vorobyov [3] where this ring was taken to be the set of integers . O. Wyler[5] also worked with such a sequence over a particular commutative ring with identity. Classification of all finite rings of order with a prime have been studied by B. Fine [7].

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Groups whose proper subgroups are Chernikov-by-Baer

Nadir Trabelsi

If $\mathfrak{X}$ is a class of groups, then a group G is called a *minimal non-$\mathfrak{X}$-group* if it is not an $\mathfrak{X}$-group but all its proper subgroups belong to $\mathfrak{X}$. We will denote the minimal non-$\mathfrak{X}$-groups by $MN\mathfrak{X}$. Many results have been obtained on $MN\mathfrak{X}$-groups, for several choices of $\mathfrak{X}$. In particular, in [1] a complete description of locally nilpotent $MN\mathfrak{N}$-groups having a maximal subgroup is given, where $\mathfrak{N}$ is the class of nilpotent groups. These groups are metabelian Chernikov p-groups, for some prime p, and hence hypercentral. Later in [2], locally nilpotent $MN\mathfrak{N}$-groups without maximal subgroups were studied and it was proved, among many results, that they are countable p-groups in which every subgroup is subnormal. In [3], it is proved that locally graded $MN\mathfrak{R}\mathfrak{N}$-groups are precisely the locally nilpotent $MN\mathfrak{N}$-groups without maximal subgroups, where $\mathfrak{R}$ denotes the class of groups of finite rank.

In this note we study $MN\mathfrak{C}\mathfrak{N}$-groups, where $\mathfrak{C}$ is the class of Chernikov groups and we prove that locally graded $MN\mathfrak{C}\mathfrak{N}$-groups are precisely the locally nilpotent $MN\mathfrak{N}$-groups without maximal subgroups and hence they are $MN\mathfrak{R}\mathfrak{N}$-groups.

In [4] a characterization of locally graded $MN\mathfrak{B}$-groups, where $\mathfrak{B}$ is the class of Baer groups, is given in terms of $MN\mathfrak{N}$-groups.

Here we also study $MN\mathfrak{C}\mathfrak{B}$-groups and we prove that locally graded $MN\mathfrak{C}\mathfrak{B}$-groups are locally finite and coincide with the normal closure of an element and that locally nilpotent $MN\mathfrak{CB}$-groups are precisely $MN(\mathfrak{D} \cap \mathfrak{C})\mathfrak{B}$-groups, where $\mathfrak{D}$ denotes the class of divisible abelian groups.

Recall that G is called a Baer group if all its cyclic subgroups are subnormal in G, and that G is of finite rank if there exists a positive integer r such that every finitely generated subgroup of G can be generated by r elements. Also G is said to be locally graded if every non-trivial finitely generated subgroup has a non-trivial finite image.

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Hilbert-Kunz function and Hilbert-Kunz multiplicity

Vijaylaxmi Trivedi

In this talk we recall the notions of characteristic p invariants of a ring (commutative and Noetherian), namely Hilbert-Kunz function and Hilbert-Kunz multiplicity. They seem an analogue of the Hilbert-Samuel function and the classical multiplicity. Here we concentrate on HK multiplicity, which is a more subtle invariant of a ring (compare to classical multiplicity): It is related to characteristic p features of the singularities of the ring. Since standard techniques, used to study the multiplicity, such as reduction, induction on the dimension etc., do not work, very few examples of the HK multiplicity have been computed so far. Here we will give an overview of results and known computations of the HK multiplicity and discuss some open problems.

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On invariants of towers of function fields over finite fields

Seher Tutdere

Let $\mathcal{F} = (F_n)_{n\geqslant 0}$ be a tower of function fields over a finite field $\mathbb{F}_q$, and let $r \geqslant 1$ be an integer. Then the limit

$$\beta_r(\mathcal{F}) := \lim_{n\to\infty} (\text{the number of places of } F_n/\mathbb{F}_q \text{ of degree } r\ )/(\text{genus of } F_n)$$

exists, and it is called an invariant of the tower $\mathcal{F}$. In this talk we describe a method for constructing towers over any finite field with many prescribed invariants being positive. Such towers are useful to obtain both good algebraic geometric codes and bounds for multiplication complexity in finite fields. They also have large asymptotic class number. Our method is based on explicit extensions of function fields. Moreover, we give some examples of recursive towers with various invariants being positive.

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Rad-Discrete Modules

Ergül Türkmen

(This is joint work with Yıldız Aydın) Throughout this study, all rings are associative rings with unity and all modules are unital left modules unless indicated otherwise. Let R be such a ring and M be an R-module. A submodule S of M is called *small* in M, denoted by $S << M$, if $S + L \neq M$ for every proper submodule L of M. For a module M, $Rad(M)$ indicates the radical of M. By a *supplement* of N in M we mean a submodule V which is minimal in the collection of submodules L of M such that $M = N + L$. Equivalently, $M = N + V$ and $N \cap V << V$ [9]. It is clear that supplement submodules are a generalization of direct summands. If $M = N + V$ and $N \cap V \subseteq Rad(V)$, then V is called a *Rad-supplement* of N in M [8]. Under given definitions, we have the following implication on submodules:

$$\text{direct summand} \Longrightarrow \text{supplement} \Longrightarrow \text{Rad-supplement}$$

As generalizations of the notion of semisimple modules, one calls a module M *(Rad-) supplemented* if every submodule has a (Rad-) supplement in M (see [9], [2] and [3]). On the other hand, a module M is called *(Rad-) $\oplus$-supplemented* if every submodule has a (Rad)-supplement that is a direct summand of M (see [7] and [5]).

A module M is called π-*projective* if for every two submodules U and V of M with $M = U + V$, there exists a homomorphism $f : M \longrightarrow M$ with $f(M) \subseteq U$ and $(1 - f)(M) \subseteq V$ [9].

It is well known that a π-projective module M is supplemented if and only if it is $\oplus$-supplemented.

For a module M, consider the following conditions:

(D_1) For every submodule N of M there exists a direct summand L of M such that $M = L \oplus K$, $L \subseteq N$ and $N \cap K << K$.

(D_2) If N is a submodule of M such that $\frac{M}{N}$ is isomorphic to a direct summand of M, then N is a direct summand of M.

(D_3) For every direct summands K and L of M with $M = K + L$, $K \cap L$ is a direct summand of M.

Every module M with the property (D_2) satisfies the property (D_3).

A module M is called *discrete* if M satisfies these properties (D_1) and (D_2) [7]. This is equivalent to M is $\oplus$-supplemented, π-projective and satisfies the property (D_2). The module M is called *quasi-discrete* if it satisfies these properties (D_1) and (D_3) [7]. We know that M is quasi-discrete if and only if it is $\oplus$-supplemented and π-projective. The concept of these modules are extensively studied by many authors.

The aim of this talk is to generalize (quasi-) discrete modules to (quasi-) *Rad*-discrete modules. In particular, we obtain various properties and characterizations of such modules.

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Gröbner-Shirshov Bases for Affine Weyl Group of Type $\widetilde{A}_n$

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This is a joint work with Dr. Erol Yılmaz.

Gröbner-Shirshov bases and normal form of the elements were already found for the Coxeter groups of type A_n, B_n and D_n in [1]. They also proposed a conjecture for the general form of Gröbner-Shirshov bases for all Coxeter groups. In [2], an example was given an to show that the conjecture is not true in general. The Gröbner-Shirshov bases of the other finite Coxeter groups are given in [3] and [4]. This paper is another example of finding Gröbner-Shirshov bases for groups, defined by generators and defining relations. We deal with the affine Weyl group $\widetilde{A}_n$ which is an infinite Coxeter group. Using defining relations, we able to find the reduced Gröbner-Shirshov bases of $\widetilde{A}_n$ and classify all reduced words of the affine Weyl group $\widetilde{A}_n$.

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The strong cell decomposition property for weakly o-minimal structures

Roman Wencel

A linearly ordered first order structure $\mathcal{M} = (M, \leqslant, \ldots)$ is called *weakly o-minimal* if every subset of M, definable in $\mathcal{M}$, is a finite union of convex sets.

Weakly o-minimal structures in general appear to be much more difficult to handle than the o-minimal ones. The main problem is that they lack so called finiteness properties, and therefore one cannot expect a reasonable cell decomposition for sets definable in them. As weak o-minimality is not preserved under elementary equivalences, one introduces a notion of a weakly o-minimal theory, that is a first-order theory whose all models are weakly o-minimal structures. Although in a model of such a theory one can prove a weak form of cell decomposition, the topological dimension for definable sets does not behave as well as it does in the o-minimal setting. For example, it does not satisfy the addition property. One can, for instance, define a set of dimension one whose projection onto some coordinate has infinitely many infinite fibers.

A class of weakly o-minimal structures in which one can smoothly develop an o-minimal style description of definable sets was considered in [MMS]. The authors prove that sets definable in weakly o-minimal expansions of ordered fields without non-trivial definable valuations are finite unions of so called strong cells, which are constructed more or less as cells in the o-minimal setting. It turns out that this result can be generalized to certain weakly o-minimal expansions of ordered groups. It was proved in [We1] that every weakly o-minimal expansion of an ordered group without a non-trivial definable subgroup has the strong cell decomposition property (SCDP). This paper also shows that every weakly o-minimal structure $\mathcal{M}$ with SCDP has some canonical o-minimal extension $\overline{\mathcal{M}}$.

During my talk I am going to discuss further properties of weakly o-minimal structures with SCDP. One of the basic results is that SCDP for weakly o-minimal structures is preserved under elementary equivalences. Combining this with some relativization of the construction of the canonical o-minimal extension, we obtain a covariant functor between the category of elementary embeddings of $\mathcal{M}$ to the category of elementary embeddings of $\overline{\mathcal{M}}$.

It turns out that in the context of weakly o-minimal structures with SCDP, definable sets (functions, relations) share many properties of sets definable in o-minimal structures. One can for instance generalize classical results of Speissegger from [Sp] concerning fiberwise properties of definable sets and functions.

An obstacle that one faces when working with weakly o-minimal structures is that the usual notion of definable connectedness does not work properly. Namely, the number of definably connected components of a definable set in general is infinite, even assuming SCDP. Nevertheless, it could be shown that equivalence relations definable in weakly o-minimal structures with SCDP share several typical properties of equivalence relations definable in the o-minimal context. These lead to the conclusion that if $\mathcal{M} = (M, \leqslant, \ldots)$ is a weakly o-minimal structure with SCDP and one of the conditions (a), (b) holds:

(a) for any $B, C \subseteq M$, B and C are independent over $\mathrm{dcl}(B) \cap \mathrm{dcl}(C)$;

(b) for any $A \subseteq M$, $\mathrm{dcl}(A)$ is an elementary substructure of $\mathcal{M}$;

then $\mathcal{M}$ admits elimination of imaginaries.

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Envelopes, Weakly Prime and Weakly Primary Ideals

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Throughout this paper all rings are commutative with identity and all modules are unitary.

A *primary decomposition* of a submodule N of M is representation of N as a intersection of finitely many primary submodule of M. Such a primary decomposition $N = \cap_{i=1}^{n} Q_i$ with p_i-primary submodules Q_i is called minimal if p_i's are pairwise distinct and $Q_j \not\supseteq \cap_{i \neq j}$ for all $j = 1, \ldots, n$.

The different definitions of a radical of an ideal in rings can be generalize to two distinct concepts in module theory. One is the the radical of a submodule which is defined to be intersection of all prime submodules containing the submodule. The other is the envelope of a submodule. If N is an submodule of an R-module M, then the envelope of N in M is defined to be the set

$$E_M(N) = \{rm : r \in R, m \in M \, and \, r^k m \in N \, for \, some \, k \in \mathbb{Z}^+\}.$$

Since $E_M(N)$ is generally not a submodule, $\langle E_M(N) \rangle$ denote the submodule generated by the envelope. Although there are some results about computation of radical of a submodule(see [5] and [7]), there is no such a result for the envelope. We give a formula for the computation of $\langle E_M(N) \rangle$ if a minimal primary decomposition of N is known. We use the concepts and results of [4] in this section.

A proper submodule N of an R-module M is called *a weakly prime submodule* if for each $m \in M$ and $a, b, \in R$; $abm \in N$ implies that $am \in N$ or $bm \in N$. A proper submodule N of an R-module M is called *a weakly primary submodule* if $abm \in N$ where $a, b, \in R$ and $m \in M$, then either $bm \in N$ or $a^k m \in N$ for some $k \geqslant 1$. The weakly radical of a submoduled N of M, denoted by $wrad_M(N)$, is the intersection of all weakly prime submodule containing N. The concepts of weakly prime and weakly primary submodules are introduced a few years ago and they have been studied by some authors (for example see [1], [2] and [3]). In section 2, we give an example to show the conjucture given in (see [4]) is false. Using the envelope of a submodule, we give a condition for a submodule to be written as an intersection finitely many weakly prime ideal.

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Posters

Leibniz algebras in characteristic p

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We give definition of restrictness for Leibniz algebras in characteristic p. We prove that cohomologies of Leibniz algebras with coefficients in irreducible module is trivial, if module is not restricted. The number of irreducible antisymmetric modules with nontrivial cohomology is finite. Leibniz algebra is called simple, if it has no any proper ideal except ideal generated by squares of its elements. We describe simple Leibniz algebras with Lie factor isomorphic to 3-dimensional simple Lie algebra sl_2 and p^m-dimensional Zassenhaus algebra $W_1(m)$.

This is a joint work with Askar Dzhumadil'dev.

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$\oplus$-Co-coatomically Supplemented and Co-coatomically Semiperfect Modules

Serpil Güngör

M will mean an R-module where R is an arbitrary ring with identity. A module M is called coatomic if every submodule is contained in a maximal submodule of M. A proper submodule N of M is called co-coatomic if M/N is coatomic. A module M is $\oplus$-co-coatomically supplemented if every co-coatomic submodule of M has a supplement that is a direct summand of M.

The following example shows that arbitrary direct sum of $\oplus$-co-coatomically supplemented module need not be $\oplus$-co-coatomically supplemented.

Example 1. *Let R denote the ring $K[[x]]$ of all power series $\sum_{i=0}^{\infty} k_i x^i$ in an indeterminate x and with coefficients from a field K which is a local ring. Then any direct sum of R, i.e. R-module $R^{(N)}$ is not $\oplus$-co-coatomically supplemented module since although $RadR^{(N)}$ is co-coatomic* *submodule of R-module $R^{(N)}$, it does not have a supplement in R-module $R^{(N)}$.*

Proposition 2. *For any ring R, any finite direct sum of $\oplus$-co-coatomically supplemented R-modules is $\oplus$-co-coatomically supplemented.*

M is called $\oplus$-supplemented if every submodule of M has a supplement that is a direct summand of M. A ring R is left perfect if and only if $R^{(N)}$ is a $\oplus$-supplemented (see [2]).

Theorem 3. *A ring R is left perfect if and only if $R^{(N)}$ is a $\oplus$-co-coatomically supplemented R-module.*

M is called co-coatomically semiperfect if every coatomic factor module of M has a projective cover. For a projective module M, M is semiperfect if and only if M is $\oplus$-supplemented (see [1]). Similarly, for a projective module M, M is cofinitely semiperfect if and only if M is $\oplus$-cofinitely supplemented (see [3]).

Proposition 4. *Let M be a projective R-module. Then M is co-coatomically semiperfect module if and only if M is $\oplus$-co-coatomically supplemented module.*

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On the minimum distance of cyclic codes

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Estimation of the minimum distance of cyclic codes is a classical problem in coding theory. Using the trace representation of cyclic codes and Hilbert's Theorem 90, Wolfmann found a general estimate for the minimum distance of cyclic codes in terms of the number of rational points on certain Artin-Schreier curves. In this poster, we try to understand if Wolfmann's bound can be improved by modifying equations of the Artin-Schreier curves by the use of monomial and some nonmonomial permutation polynomials. Our experiments show that an improvement is possible in some cases.

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Torsion free covers in the category of representations of quivers

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Joint with and Sergio Estrada.
Let R be a ring, $R\text{-}\mathcal{M}od$ be the category of left R-modules and Q be a quiver (i.e. a directed graph). We prove the existence of torsion free covers, relative to a torsion theory, in the category of representations by modules of a quiver, denoted by $(Q, R\text{-}\mathcal{M}od)$, for a wide class of quivers provided that any direct sum of torsion free and injective R-modules is injective.

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Localization of supplemented modules

Esra Öztürk

Let R be a commutative ring with identity and M be an $R-$module. We present the relation between an $R-$module M and an R_P- localization module M_P in the view of being supplemented for all P maximal ideals of R.

This is joint work with Şenol EREN, Ondokuz Mayıs University

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Fibonacci-Gcd Matrisleri

Mehmet Sever

$S = \{x_1, x_2, ...x_n\}$ pozitif tamsayıların sıralı bir kümesi olsun.

$$(S) = (s_{ij}) = (x_i, x_j)\,, n \times n$$

matrisine S kümesi üzerinde GCD MATRİSİ denir. Pozitif tamsayılardan tanımlı ve kompleks sayılar kümesi değerli her fonksiyona aritmetik fonksiyon denir. Örnek olarak Euler'in Phi fonksiyonu φ, Riemann'ın zeta fonksiyonu ζ, Mobius'un mü fonksiyonu μ verilebilir. Literatürde çalışılmış matrislerin başlıcaları $(S) = (s_{ij}) = (x_i, x_j)$, $\left(\frac{1}{S}\right) = \left(\frac{1}{(x_i,x_j)}\right)$ ve $(S^n) = (x_i, x_j)^n$ matrisleridir. Burada matrisleri teşkil etmedeki en büyük iş matrisi meydana getiren aritmetik fonksiyonu bulmaktır. GCD matrisini φ fonksiyonu belirlemektedir. Biz bu çalışmamızda GCD matrisinin her girdisine bir ekleyip karşılık gelen Fibonacci dizisi değerini aldık.

$$(S) = (s_{ij}) = (x_i, x_j) \to F_{(S)} = F_{(x_i,x_j)+1}$$

ve bu matrisi üreten aritmetik fonksiyonu, bu matrisin determinanatını ve yapısını inceledik.

$$\sum_{d|n} g\,(d) = F_{(x_i,x_j)+1}$$

Burada bir ekleme sebebimiz meydana gelen matrisin tekilliğini kaldırmaktır.

Benzer düşüncelerle

$$\sum_{d|n} f(d) = (-1)^n F_n$$

$$\sum_{d|n} h(d) = \frac{F_{n+1}}{F_n}$$

aritmetik fonksiyonları yardımı ile sırasıyla alterne-fibonacci matrisi ve reciprocal fibonacci matrisi meydana gelmektedir. Keza S kümesi pozitif tamsayıların bir kümesi olmak üzere bu küme üzerinde bir kısmi sıralama bağıntısı($\preceq$) tanımlayıp

$$\sum_{x_i \preceq x_j} h(x_i) = F_{x_i} + F_{x_j}$$

şeklinde bir aritmetik fonksiyon daha tanımlayıp kümeyi ve üzerindeki işlemi genelledik.

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On (cofinitely) generalized amply weak supplemented modules

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Let R be a ring and M be a left $R-$module. In this paper, we will study some properties of (cofinitely) generalized amply weak supplemented modules (CGAWS) as a generalization of (cofinitely) amply weak supplemented and give a new characterization of semilocal rings using CGAWS-modules. Nevertheless, we will show that (1) M is Artinian if and only if M is a GAWS-module and satisfies DCC on generalized weak supplement submodules and on small submodules. (2) A ring R is semilocal if and only if every left R-module is CGAWS-module.

This is joint work with Şenol EREN, Ondokuz Mayıs University

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